MATH MAGAZINE

FORALL

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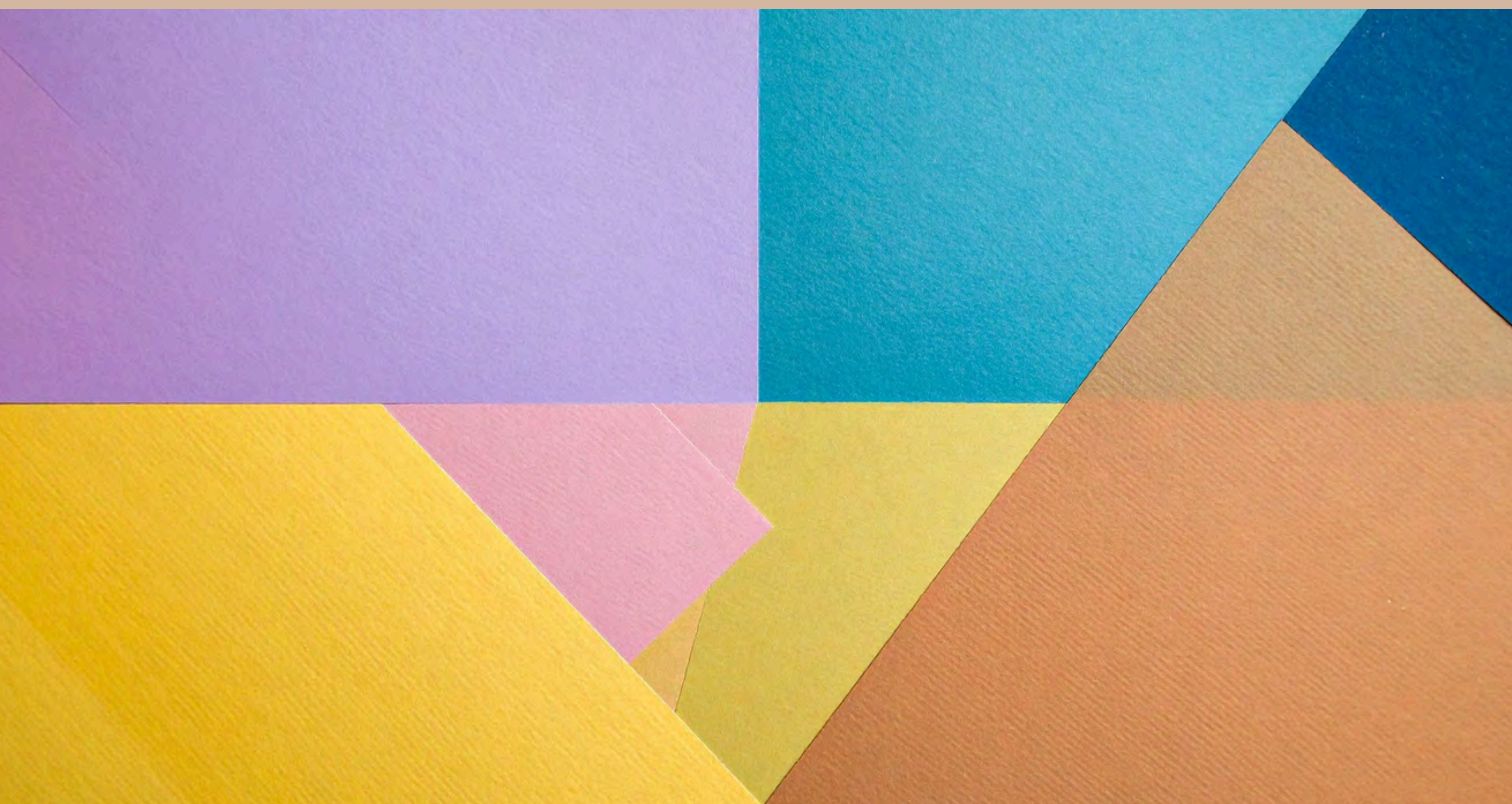

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Cover design by Özlem Uğurlu, created in Canva.

Welcome to *For∀LL*

Welcome to this new issue of For∀LL! We are excited to introduce two new members of our social media team, Maya Golebiowski and Taylor Pike. We are happy to have you with us.

This issue covers a range of topics across mathematics. We explore Beatty sequences and their connections to music, and take a journey from integer partitions into Young tableaux. We also look at phase retrieval in imaging and signal processing, where randomness in measurements turns a difficult problem into one that is well-behaved and solvable. We also hear from undergraduates who attended a mathematics conference, where they share their personal experiences of growing as mathematicians. Last but not least, enjoy our new Math in the Wild challenge and a new crossword puzzle created for our magazine!

Our goal has always been to make mathematics accessible to everyone. We warmly welcome submissions from undergraduate and graduate students and researchers alike, whether it is a class project or original research; we value the richness each perspective brings to our mathematical community. If you have something to share, we would love to hear from you.

Thank you for being a part of our journey, and we hope you find inspiration and enjoyment in the pages that follow.

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Interview Series

Interviewing: Dr. Christy García

Interviewed by: Nathaniel Alloway, Master's Student, Department of Mathematics and Statistics, Saint Louis University
Evan Socher, Undergraduate Student, Department of Mathematics and Statistics, Saint Louis University

Dr. García is an associate professor of Spanish with SLU's department of linguistics, languages, and cultures. Her research is in Hispanic linguistics, focusing on sociophonetics. Her work involves a number of interesting statistical techniques, which are expounded on below. She is also a collaborator of SLU faculty member Dr. Darrin Speegle from the department of mathematics and statistics, and has supervised capstone projects in both computer science and data science.

This interview has been condensed and edited for clarity.

Interview

What is sociophonetics?

Sociophonetics is the study of sounds and what they mean to people. The *phonetics* part is the study of sounds — their acoustic and articulatory properties. But then the *socio-* part comes in when we look at the social meaning of sounds. To give an example, vowel sounds in English can be really indicative of where a speaker is from. So, the way someone might say, "Dontcha know?" might tell you that they're from around Minnesota.

Sociophoneticians can either look at the production or perception of sounds, and I do both in my research. For production, I look at the patterns of sounds in a community to see whether they correlate with social factors like age or gender. For perception, I play audio files of speech to participants and ask them things like how old they think the speaker is or what level of education they may have.

How did you first get into sociophonetics?

How I got into *linguistics* — I have always really liked language, but I also really liked math. Actually, I was originally a math and Spanish double major in undergraduate. I then stumbled upon linguistics, which is basically a combination of language and math (e.g. by way of statistics and predictive models). Linguistics isn't a thing you can study in high school, so I discovered it once I got to university. Once there — no offense — I dropped the math major because what I discovered I liked about math was its application to language.

I think it makes sense what I ended up doing because a lot of math is about looking at patterns and finding patterns, and that's what I'm doing in my research: I'm looking into patterns in the way people speak and the way people think about language.

I discovered during my undergraduate that I was most interested in sounds and what they mean to people. When I studied abroad in Argentina, it was one of the first times where I thought about sounds from that social point of view. In the Spanish of Buenos Aires, the *y*'s and *ll*'s can be pronounced as [ʃ] or [ʒ][1], with the specific pronunciation depending on where in Buenos Aires you grew up in, age, social class, etc. Being there, I could see the "social life" of sounds, if you will, and that experience is what got me interested in that specific subset of linguistics.

What did your career journey look like afterward?

For a while, I wanted to teach both math and Spanish at high school. I don't actually know if you can get certified in both subjects simultaneously, but that was what I was going to try to do. But throughout my undergraduate coursework and after my thesis project, I discovered I really enjoyed doing research, so that initial idea morphed from wanting to be a teacher into wanting to be a professor. I ended up pursuing a Master's and then a Ph.D. in Hispanic linguistics.

Little by little, I discovered a passion for higher ed. I really liked the combination of being able to work with students, travel and do research, present at conferences, and conduct data analysis. So that's how I ended up here.

What, specifically, does your research look like?

I'm looking at different varieties of Spanish and sounds that are variable. Variations in those sounds may mark regional differences, or generational differences, or gender differences, etc. I've done research mostly in Argentina and Ecuador, but I've also done a project on Puerto Rican and Mexican Spanish.

Unlike in English, where vowel sounds are a lot more varied, in Spanish, it's mostly the consonants that vary. In the project where we compared Puerto Rican and Mexican Spanish, we looked at the pronunciation of *s* at the ends of syllables. In Puerto Rico, it's very common to aspirate or omit the *s* entirely (which you might notice in Bad Bunny's music!), whereas that's not a feature of Mexican Spanish. Speakers often consider pronouncing the *s* as "classier" — it's "standard" or normative.

We wanted to know whether listeners took into account where a speaker is from when determining whether speech was "classy" — is it "classy" across the board to pronounce the *s*, or are expectations different when a speaker is perceived as Puerto Rican? We ended up finding that listeners *do* take this into account; they attenuate the social meaning that they might apply based on where the speaker is from.

In a different study, we looked the pronunciation of *s* between vowel sounds in Ecuadorian Spanish. Normally, when an *s* is between vowels, it's pronounced as [s], but sometimes it can be pronounced as [z] with the vocal folds vibrating. There, we found that male speakers tended to use [z] more than women.

When you're testing for perceived variations in these sounds, do you usually tell the listener what sounds you're trying to test for?

No — we're trying to test for listeners' subconscious reactions. To do this, I use a technique called a *matched-guise test*, where you take the same person, saying the same sentence, and the only thing you manipulate is the variable that you're looking at. This keeps the context and everything else the same. The listeners think they're listening to all different people — you space out the matching samples so that they don't remember they've heard the same person before — and that way you can compare how listeners rate samples based on the variables of interest.

How do you do your data analysis?

If I'm looking at the production of sounds, I record interviews with native speakers, and then I do an acoustic analysis. I look at a *spectrogram*[2], and then I measure some aspect of the speech of the participants. That usually gives me some sort of continuous variable. For example, in the case of [s] versus [z], I can measure the *voicing* of the sound, i.e. how much the vocal folds are vibrating, giving a measure anywhere from 0% to 100%. Based on the dependent measures, I make a *mixed-effects model* that looks at age, gender, other social factors, and the relevant linguistics features (e.g. what's the preceding vowel? is the syllable stressed?). The model lets me see which factors are significant and can explain the variation.

I have had to get a little creative with the modeling. Typically, if a linguist is looking at a continuous variable, they use a linear regression model. That

[1] The [ʃ] sound is the *sh* in *shush*. The [ʒ] sound is "*zh*" or the *s* in *vision*.

[2] See vol. 2, no. 2 for an article on spectrograms.

doesn't work for my data though as they're not normally distributed. So, I use two different methods. I use *inflated beta regression*, which is designed to handle increased proportions near 0% and 100%. This is what my percent voicing data look like. I then also do *ordered logistic regression* for three different categories of voicing (none, partial, and complete).

For perception studies, these are mostly online studies using *Likert scales* — e.g. rate this person on a scale from low education to high education. I then convert the results to numerical data, center, and standardize. Afterward, I usually do *factor analysis* because social characteristics are often patterned together. You could think of, say, a bigger "status" factor that incorporates education level, rurality, etc. So, I look at the dependent measures and see whether any are patterning together to decide whether I want to make any joint factors.

When people rate using a Likert scale, people usually stick to the middle, so we typically design our scales as a *six*-point scale. Participants have to pick one side, but still tend to stay near the middle, so our data there end up roughly normal.

How did you get involved with data science and computer science people on campus?

I've worked with Darrin Speegle (math/stats), Flavio Esposito (computer science), and Kevin Scannell (formerly math/comp. sci.) in the past, and I've also worked with computer science capstone students. This year, I'm working with four data science capstone students. I've given them a raw Qualtrics dataset for another matched-guise study and asked them to clean it and look for patterns. They've done some basic visualizations and are now working on factor analysis. I've done these projects before, so I have an idea of what I would do, but it's interesting to see their perspective on what *they* would do and what questions are interesting for them. It's useful in that it gives me some questions that I can now ask on my own, and it's also fun to nerd out with them.

There's now a new minor in linguistics, which will hopefully lead to more opportunities to do data science collaborations.

Switching tracks, have you faced any challenges in your work — research, teaching, and so forth?

On the research side, it's kind of hard to be an expert in all the things one needs to be an expert in — or to have access to all the knowledge bases one might need. In my field specifically, we draw methods from sociology, linguistics, and statistics. The breadth of knowledge one needs to have can be challenging. I've had to learn some new things on my own, so I'm fortunate to have a tight-knit community from both here and from grad school.

On the more general professional side, we tend to wear a lot of hats. Students may only see us teach in the classroom, but it's only like one of ten things we do in a week. The balancing of teaching, research, service — the gazillion committees that we're on — the small administrative things — it's challenging to juggle all of those things and switch from one to the other. But at the same time, it's one of the things I enjoy the most about my job. There's never a dull moment.

There can also be extreme peaks and valleys for how much work you have going on, which can be unpredictable. For example, you can send in an article, have it be under review for four months, and then all of a sudden they get back to you and say, "We need revisions in two weeks" — and it's completely unpredictable! So again, it's one thing I appreciate about my job despite finding it challenging.

You mentioned previously needing to be an expert in many areas and the connections that you've had with other professionals. Is it common to collaborate in your area, and how do you go about building community?

Yes, it's common in my subfield of linguistics. But it's interesting being in a humanities department, where's that not the norm. There, it's more common to have single-author papers, and usually one publishes books instead of articles. So it's interesting navigating being in a humanities department while being a social scientist.

But yeah, I think that collaboration is super beneficial for many reasons. Everyone has their strengths and weaknesses as a researcher. I love collecting data, I love analyzing it, but I hate writing a lit review. And I have collaborators that love the writing part and hate the data analysis part, so it works out very well to play to everyone's strengths — you don't have to be an expert in everything. (Actually, it's somewhat common for linguists to co-author papers with statisticians.)

A lot of people I collaborate with are people I went to grad school with as we work in the same area of sociolinguistics. Conferences are another place where I've met a lot of my research community. One thing that was initially challenging about SLU was that there wasn't a linguistics department. Linguists were siloed in different departments (e.g. speech, language, and hearing sciences). Now we have a cohort in the department, which has helped with building community locally.

On the topic of socialization again, in what ways have social bias appeared in your work?

A lot of times, particularly in perception, studies can reveal discrimination. I forget which linguist has said this, but language is like the last acceptable form of discrimination. We feel like we can tell people what to do with their language, whereas with other types of discrimination we go, "Oh no, that's wrong" — we can't tell people they *have* to do this or that with who or how they are. Linguistic discrimination is somehow permissible even though, at the end of the day, the linguistic features that are discriminated against often correlate with racial, gender, and other social features.

As one purpose of this line of research, we want to give voice to all the different varieties of language and show that they're all equally structured and useful forms of communication, but doing this work doesn't immediately change perceptions.

Personally, I can sometimes feel non-native speaker bias when conducting interviews for research, although sometimes that can be beneficial as I can frame investigations as wanting to learn about the participant's culture.

What does everyday life look like for you, outside being a professor?

Every day is different — again the multiple hats. In terms of hobbies, I love to sing. I was part of the St. Louis County Community Chorus, but I've had to put that aside this semester due to time conflicts. I'm also a part of this group of women who meet at Tower Grove Park every Thursday morning, rain or shine or cold, for exercise. I'm a big foodie: My colleague Ander Beristain says that I'm better than Yelp. I also have my own espresso machine at home.

I also love to travel. It's one of the reasons why I ended up doing what I do. As an undergraduate, I saw my professors going to different countries to collect data and go to conferences. So I thought, hm, getting my travel paid for… Yes! I like to talk to people, and the specific research that I'm doing allows me to travel and allows me to talk to people.

What's one piece of advice you can give to undergraduate readers of the magazine?

It might sound a little cheesy, but — just being okay with not having everything figured out. I'm pretty atypical in that my trajectory was pretty much linear. It's not usually like that, right? Amongst my family, friends, and colleagues, the professional trajectory is usually a lot more windy. You study one thing, and it gets you into another thing, and then you're interested in this, etc. A lot of students come into college with blinders on — I want to do X, I've wanted to do X forever, don't talk to me about anything that's not X. So taking the blinders off, it's good to be open to discovering new fields and not knowing what's next. To use a Jesuit word here, sometimes your *vocation* comes at you from a backwards way, or from an experience you were not necessarily expecting.

Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called "EvenQuads" to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.

Ivelisse Rubio (b. 1962)

Rubio is the director of the Laboratorio Emmy Noether, a research lab promoting diversity in science and computational math, and professor of computer science at the University of Puerto Rico at Río Piedras. Her research interests are in computational algebra, finite fields, and coding theory. Rubio co-founded the SIMU REU at the University of Puerto Rico at Humacao and the MSRI-UP REU in Berkeley CA, serving as director of each. Her awards include the inaugural AMS Programs that Make a Difference Award (2006), the SACNAS Presidential Service Award (2006), and the IPC Falconer Award for Mentoring and Commitment to Diversity (2010). Rubio is an editor of *The American Mathematical Monthly*.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.

EvenQuads

Beatty Sequences and Infinite Music

By Zane Davidson

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Overture

In 2002, Scottish electronic music duo Boards of Canada released their sophomore studio album, Geogaddi. The second track, entitled "Music is Math", posits a conclusion far from the popular conception of the medium. Music enjoys a seat among the pantheon of the arts in the collective imagination, a member of the more ephemeral and less definite category of human creation. The title of the song seems at first tongue-in-cheek, a simple antithesis for dramatic effect.

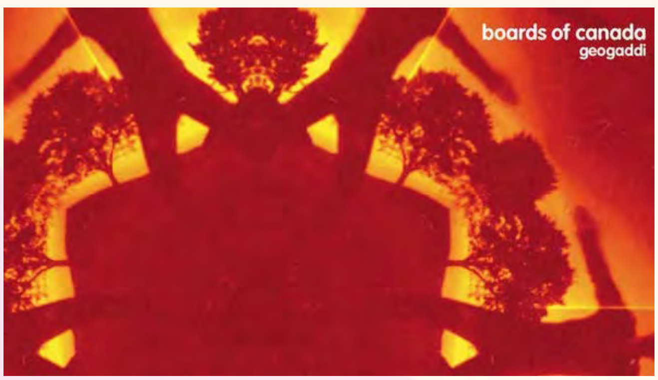

Figure 1: *Boards of Canada, Geogaddi.*

But in reality, they were simply stating a fact gleaned from the reality of their work. Music is math, and has been known to be described by mathematical principles since its essential conception. However, arguably the first to truly study the relationship between natural numbers and harmony were the Pythagoreans, who laid the groundwork for modern musical theory. They were the first to express musical scales (the progressions between one note and its octave) as ratios of integers. Plato remarks on their efforts—though somewhat critically—in *The Republic*: "...I am referring to the Pythagoreans, of whom I was just now proposing to enquire about harmony. For they too are in error, like the astronomers; they investigate the numbers of the harmonies which are heard, but they never attain to problems—that is to say, they never reach the natural harmonies of number, or reflect why some numbers are harmonious and others not" [2].

Later, Euclid, known for his *Elements*, which systematically compiled and proved not only Pythagoras's theorem but a vast body of mathematical knowledge of the time, continued this legacy (albeit indirectly) with his Euclidean algorithm, a method for finding the greatest common divisor of two integers. Euclidean rhythm, which is used ubiquitously today, especially in electronic music, utilizes Euclid's algorithm to distribute k beats as evenly as possible within n time steps. As shown by Godfried Toussaint in 2005, it can describe many common cyclic phrases (called ostinatos) that serve as a base for a large family of polyrhythmic textures in Sub-Saharan African rhythm and world music in general, see [4]. Music is math as much as it is art. While modern music theory is non-axiomatic, musicality can be described mathematically, through acoustics, and attempts to describe the properties of music have involved abstract algebra, set theory, and number theory, the last with which I am concerned in this paper.

Sonata: Beatty's Question

In 1926, Samuel Beatty, professor of mathematics at the University of Toronto, proposed problem 3173 in *The American Mathematical Monthly* [1]:

"If X is a positive irrational number and Y its reciprocal, prove that the sequences

$$(1+X), 2(1+X), 3(1+X), \ldots$$
$$(1+Y), 2(1+Y), 3(1+Y), \ldots$$

contain one and only one number between each pair of consecutive positive integers."

His question, and the subsequent proofs in reply, would lead to the conception of the Beatty sequence and its properties.

To illustrate the thrust of question 3173, let $X > 0$ be irrational and $Y = 1/X$. Consider the sequences $a_n = n(1+X), b_n = n(1+Y)$ with $n \in \mathbb{Z}^+$. For each pair of consecutive positive integers $(k, k+1)$, exactly one of the sequences has a term in that interval.

To show this, we first determine when a term falls in $(k, k+1)$ at all, then show exactly one term across both sequences does. A term a_n lies in the interval $(k, k+1)$ if and only if

$$k < n(1+X) < k+1 \iff \frac{k}{1+X} < n < \frac{k+1}{1+X}.$$

Since $X > 0$, we have $\dfrac{1}{1+X} < 1$, so this open interval has length strictly less than 1. Hence, it can contain at most one integer. Similarly, for its reciprocal,

$$b_n \in (k, k+1) \iff \frac{k}{1+Y} < n < \frac{k+1}{1+Y}.$$

Since $Y = 1/X$, this interval has length $\dfrac{1}{1+Y} = \dfrac{X}{1+X}$. So each interval contains at most one integer.

It remains to show that exactly one of the two intervals contains an integer. To show this, let

$$\alpha = \frac{1}{1+X}, \quad \beta = \frac{1}{1+Y} = \frac{X}{1+X} = 1 - \alpha.$$

Note that since X is irrational, $\alpha = \dfrac{1}{1+X}$ is also irrational. The intervals become

$$(k\alpha,\ (k+1)\alpha), \quad (k\beta,\ (k+1)\beta).$$

Each has length strictly less than 1, so each contains at most one integer. Now (and hinting towards our minuet) take the floor:

$$k\alpha = \lfloor k\alpha \rfloor + \epsilon$$

where $0 < \epsilon < 1$ is the fractional part. Since $k \in \mathbb{Z}^+$ and α is irrational, $k\alpha \notin \mathbb{Z}$ and hence $\epsilon \neq 0$. Then the only integer that could fall inside the first interval is $\lfloor k\alpha \rfloor + 1$. This integer lies in the interval exactly when it is less than $(k+1)\alpha$. More precisely,

$$\lfloor k\alpha \rfloor + 1 < (k+1)\alpha = k\alpha + \alpha = \lfloor k\alpha \rfloor + \epsilon + \alpha$$

if and only if

$$\epsilon + \alpha > 1 \iff \epsilon > 1 - \alpha = \beta.$$

For the second interval, since

$$k\beta = k - k\alpha = k - (\lfloor k\alpha \rfloor + \epsilon) = (k - \lfloor k\alpha \rfloor - 1) + (1 - \epsilon),$$

the fractional part of $k\beta$ is $1 - \epsilon$. Since $\epsilon < 1$, $1 - \epsilon$ is nonzero and $k\beta$ is not an integer. Thus, by the same reasoning, the second interval contains an integer if and only if

$$(1 - \epsilon) + \beta > 1 \iff \beta > \epsilon.$$

Comparing the two conditions, the first interval contains an integer when $\epsilon > \beta$, and the second when $\epsilon < \beta$. These are mutually exclusive and exhaustive: equality cannot occur since

$$\beta = \epsilon \iff k\alpha + \alpha = \lfloor k\alpha \rfloor + 1 \in \mathbb{Z},$$

which is a contradiction as α is irrational. That is, for a fixed $k \in \mathbb{Z}^+$, exactly one of the intervals contains an integer. Equivalently, exactly one term from either a_n or b_n lies in the interval $(k, k+1)$.

Intermezzo: Polyrhythms

You might wonder how this proof relates to music. A reasonable doubt; however, recall that most mathematical properties of music relate to subdivisions of sequences, with the iterating element being discrete time. Allow me a short digression to explain my premise.

Many musical traditions rely heavily on polyrhythm and polymeter to create rhythmic complexity and depth. From the Western canon to jazz to traditional sub-Saharan African music, interlacing rhythms are a staple of musical composition. Polyrhythm is the simultaneous use of two or more rhythms within the same rhythmic cycle or meter, where the rhythms are played with equal subdivisions. For example, a polyrhythm like $3:2$ (read as “three over two”) consists of two rhythms: one of three beats per measure and one of two beats per measure as its counterpart.

In this paper, I focus on polyrhythm, although polymeter (dueling instruments playing different meters, their patterns repeating across different lengths of time, unlike polyrhythm) is also achievable with this technique.

Figure 2: *Simple 3:4 polyrhythm.*

Simple polyrhythms are described as periodic; they predictably "resolve" after a certain number of beats. In the case of polyrhythms like this coprime 3:2 polyrhythm (typical in modern jazz and deriving from permutations like 6:4 that are the foundation of much of sub-Saharan African musical tradition) the time to resolve can be found by finding the time it takes for all the component rhythms to align. This time (in measures) is given by taking the least common multiple of all components in the ratio and the number of beats per measure and dividing the result by the number of beats per measure. For example, $3:5$ in $\frac{4}{4}$ time is $\frac{\mathrm{lcm}(3,5,4)}{4} = 15$ measures until repetition. Such a predictable resolution is no failure by any means: incredible complexity can be achieved with these simple tools. Consider Brahms's *Intermezzo in A Major* or Chopin's *Études and Nocturnes* (Op. 9 No. 1 features an incredibly difficult 11:6). And anyway, complexity is no indicator of quality in art.

Minuet: The Beatty Sequence and Rayleigh's Theorem

The problem of how to create or reduce complexity is an invitation to think, however. This project was born of my wondering whether one could generate polyrhythmic sequences without repetition or predictable resolution. Through the literature, I found the modern formulation of the Beatty sequence, derived from the conditions of the original question. This variation is concerned with algorithmically generating the Beatty sequence of integers for a given $r \in \mathbb{R} - \mathbb{Q}$. It can be defined as follows:

For any irrational number $r > 1$, a Beatty sequence can be generated of the form $\beta_r = \{\lfloor r \rfloor, \lfloor 2r \rfloor, \lfloor 3r \rfloor, \ldots\}$. Since r is irrational, the gap between consecutive terms $\lfloor nr \rfloor$ and $\lfloor (n+1)r \rfloor$, which is either $\lfloor r \rfloor$ or $\lfloor r \rfloor + 1$ depending on the fractional part of nr, never settles into a repeating pattern, making β_r aperiodic. This definition directly proceeds from our proof above: substituting r as our irrational number, we see for any two consecutive integers k and $k+1$ that

$$k < nr < k+1 \iff \lfloor nr \rfloor = k$$

In other words, $k \in \beta_r$ if and only if $n \in \mathbb{Z}^+$ exists such that nr lies in the interval $(k, k+1)$. To illustrate, let us generate the Beatty sequence of integers for the famous golden ratio, $\phi = \frac{1+\sqrt{5}}{2}$. Take first the sequence of integer multiples nr (decimals truncated for clarity):

$$\{1.618, 3.236, 4.854, 6.472, 8.090, 9.708, 11.326, \ldots\}$$

Then take the floors:

$$\{1, 3, 4, 6, 8, 9, 11, 12, 14, 16, 17, 19, 21, 22, 24, 27, \ldots\}$$

But what about its complementary sequence? The complement is defined by $s = \frac{r}{r-1}$ so that $\frac{1}{r} + \frac{1}{s} = 1$ is satisfied. Taking $\lfloor ns \rfloor$ gives:

$$\{2, 5, 7, 10, 13, 15, 18, 20, 23, 26, 28, 31, 34, 36, 39, \ldots\}$$

Notice these sequences give a visual example of the essential property of Beatty sequences: the sequences together can "contain one and only one number" between each pair of consecutive positive integers. Since we have proven that for each pair, the sequences contain exactly one integer between them, we can conclude that the sequences β_r, β_s perfectly partition the set of positive integers. This principle is illustrated in the sequences above.

In fact, this property of sequences defined as above was documented even earlier than Samuel Beatty's question. In 1896, John William Strutt, 3rd Baron Rayleigh, an English physicist, wrote about such sequences in his book *The Theory of Sound*, [3]. Though a treatise on acoustics, it inspired me to follow this thread. The Baron shares the eponym with Samuel Beatty: the theorem is sometimes called Rayleigh's Theorem.

Crescendo: Tidal Cycles

Finally, let us employ our Beatty knowledge to compose some music! The key observation is that we can section musical measures into subdivisions and use a Beatty sequence to aperiodically demarcate when to play which notes and when to be silent, creating unpredictable rhythms. This process takes multiple steps.

First, an introduction to our musical tool. Tidal Cycles is an open-source live-coding environment for algorithmic pattern generation, written in Haskell. Don't let the intimidating language scare you, however: Tidal Cycles (referred to henceforth as simply Tidal) is all about making music. It allows the creation and live evaluation of musical patterns, passed from GHCi (the Haskell interactive interpreter) to SuperCollider (an audio synthesis program). We'll also use the Didactic Pattern Visualizer (DPV) library to extend Tidal and visualize our patterns. To begin, I wrote a function in Haskell to generate Beatty sequences.

```
beattySeq :: Double -> [Integer]
beattySeq a = map (floor . (* a) .
↪  fromIntegral) [1..]
```

In Haskell, lists are often analogous to loops in other programming languages. They are intended to be traversed and lazily evaluated. Thus, whenever this function is called and passed an irrational number a, it lazily maps our Beatty function over the positive integers, computing $\lfloor an \rfloor$ such that n is drawn from the generator $n \leftarrow [1..]$ (the list of all positive integers). We compute its complement like this:

```
comp :: Double -> Double
comp a = a / (a - 1)
```

Observe complementary behavior of Beatty patterns in [A1] and [A2]. Notice how in the former, the instruments never overlap, demonstrating their partitioning property. The point of the second is not so obvious: what we are doing is dictating which set of notes we can play on a given hit, whether main sequence or complementary. Since these patterns are being stepped through constantly, the pattern seems semi-random with a relatively evenly distributed partition like that produced by $\sqrt{2}$. However, this technique becomes useful, say, if we want to use something with a hit much more rare on the main sequence and (by the partitioning property) much more common on the complement, like $\sqrt{500}$. We could induce scarcity on a particular note, set of notes, or instrument, and reverse the trend if we then pass to the complement.

Now, since events in Tidal are outputs of a function of time, we must provide a list that will serve as a key for those outputs.

```
beatty_mask :: Double -> [Double]
beatty_mask a =
    let go i bs@(b:rest)
        | i == b = 1 : go (i+1) rest -- hit:
        ↪  allow event
        | otherwise = 0 : go (i+1) bs -- miss:
        ↪  block event
    in go 1 (beattySeq a)
```

This function takes our sequence and converts it recursively to a binary list where 0 represents an integer not contained in the sequence and 1 represents a contained integer. Finally, we control the instrument using the built-in `Take` method for indexing instrument parameters. However, this method has some serious problems. Firstly, we can only use `Take` on parameters. This means we can only simulate masking notes, as the closest we can get to not triggering an instrument on the nth cycle is by simply setting the gain to 0 on beatty_mask $[n] = 0$. This means we cannot use DPV to visualize it (no fun), and it can also cause clipping and other issues if we attempt high note density with this method. Instead, I refactored the code to directly translate the sequence into a boolean list, which can be passed to the mask function of an instrument control.

```
beattyMaskStream :: Double -> [Bool]
beattyMaskStream a =
    let go i bs@(b:rest)
        | i == b = True : go (i+1) rest
        | otherwise = False : go (i+1) bs
    in go 1 (beattySeq a)
```

In addition, now that we have a Boolean mask, we cannot simply rely on `Take` to request values from the Beatty stream. If we pass the stream directly to the mask function, it receives values as fast as the stream function to compute them, which causes interpreter errors as GHCi receives multiple mask arguments per cycle. I also realized that to create polyrhythms, I need to ensure that when two instruments are played with equal measures, they divide their beats across the measure evenly. With `Take`, they simply played on the start of each cycle (measure) aperiodically, meaning I wasn't even creating polyrhythm!

This code fixes this issue, by defining a step value, indicating the subdivisions of a measure for which to calculate a Beatty mask. The function `sig` turns continuous time into a running subdivision index k, determined by the chosen number of steps per cycle. We select the kth Boolean from the infinite Beatty mask, and apply this lookup across the entire time pattern. This way, we can sample the sequence in sync with musical time, allowing us to cast to a Tidal Pattern. Patterns in Tidal are a bespoke time-structured datatype — functions from time to events. That is why we could not directly cast the entire recursive sequence to a Pattern; they expect a time interval as input, and serve as queryable mappings from time to events.

```haskell
-- allow for in-cycle evaluation by a certain number of subdivisions (steps)
-- for example, steps = 16 evaluates the mask on each 16th note, steps = 4 evaluates at quarter notes, etc.
beattyMaskPatternSteps :: Integer -> Double -> Pattern Bool
beattyMaskPatternSteps steps a =
    (\k -> beattyMaskStream a !! k)
        <$> sig (floor . (* fromIntegral steps))

do
asap $ connectionMax 6 # speedSequenser 4
d5 $ grid "1 0!7"
d1 $ mask (beattyMaskPatternSteps 8 (sqrt 2)) $ s "bd*8" # connectionN 1
d2 $ mask (beattyMaskPatternSteps 8 (sqrt 3)) $ s "hh*8" # connectionN 2
d3 $ mask (beattyMaskPatternSteps 8 (sqrt 5)) $ s "kick:6*8" # connectionN 3
d4 $ mask (beattyMaskPatternSteps 8 (sqrt 7)) $ s "snare:3*8" # connectionN 4
d6 $ mask (beattyMaskPatternSteps 8 (sqrt 11)) $ s "perc:5*8" # connectionN 5
```

This is a constructed drum-kit playing the Beatty rhythm. The intervals between Beatty numbers grow proportionally to the size of the irrational number used to generate them. Thus, the drum samples become sparser over time, with the kick drum "bd" the most frequently triggered. The `beattyMaskPatternSteps` function is passed as a parameter to the mask function, which accepts its boolean Pattern as a lookup. I have configured it to evaluate the sequence on each eighth note. Since time is always increasing in Tidal, the combined pattern will never repeat, producing an infinite rhythm (until Double loses precision, that is). Check it out here [A4].

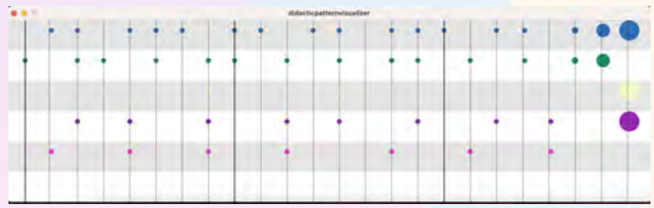

Figure 3: *DPV visualization of the Beatty drum kit.*

Observe the three measures split into eighth notes. Each colored dot is an instrument, corresponding in order to the code block above. Notice both the decreasing frequency and irregular patterns of the dots in each pattern. This is Beatty rhythm!

Finale: Improvements and Final Thoughts

While the rhythmic generator is working, some improvements are in order. Firstly, I couldn't help myself from golfing a bit, and came up with some useful simplifications and optimizations. Before, with the code `a !! k`, we requested the kth element of the linked list, requiring a linear traversal from front to index each call. This shouldn't be a problem for short runs; however, I couldn't resist an $O(1)$ solution (okay, it's still $O(N)$ for N steps, but per-query it's faster). Now, we simply compute the interval at the current timestep and perform a single comparison.

```haskell
-- check exists n in [i/a,(i+1)/a) with i = current step count
isBeatty :: Double -> Int -> Bool
isBeatty a i = floor ((fromIntegral (i+1)) / a) > floor ((fromIntegral i) / a)
-- step through cycles and create a pattern from the returned status of the cycle's membership
beattyMaskPatternSteps :: Integer -> Double -> Pattern Bool
beattyMaskPatternSteps steps a =
  (\k -> isBeatty a (k+1)) <$> sig (floor . (* fromIntegral steps))
```

So I rewrote all of it, forgoing the stream for a simple check derived from the original bounds of the sequence. If the floor of the upper bound is greater than the floor of the lower bound, the interval contains an integer, and the time step should be included. If not, it is excluded. Then simply index from $k+1$ as i proceeds from 0 when computing steps. Much faster!

CODA

Finally, check out my magnum opus, my Musical Offering, the coolest thing I have yet to come up with: [A3]. Turn the headphones down, at least initially, as the reverb can be startlingly loud. It features vocal chops triggered by a Beatty sequence, stuttering synth drives treated likewise, and my best emulation of an eerie walk home at midnight in the snow.

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Audio and Data Samples

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Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called "EvenQuads" to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.

Rodica Simion (1955–2000)

Simion was from Romania and did her graduate work in the US. She held positions at Southern Illinois University and Bryn Mawr College, and died a few months after becoming Columbian School Professor at George Washington University. Simion made significant contributions to algebraic combinatorics, including co-founding the subfield of pattern-avoiding permutations. She contributed to the combinatorial math community by co-organizing many conferences, and by founding the highly impactful Summer Program for Women in Mathematics, which ran from 1995–2009. Simion further co-organized a museum exhibit about mathematics based at the Maryland Science Museum. Together with a traveling version, it was visited by over four million people in five years.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.

EvenQuads

Phase, Probability, and Geometry: Retrieving a Signal with (Un)Certainty

By Leo Schrader

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When one thinks of random processes, one typically thinks of unpredictable systems. This association is natural: by definition, the result of a dice roll or coin toss cannot be predicted perfectly in advance. However, in this article, we will illuminate a centuries-old, well-studied, and physically motivated system for which inserting pure randomness actually induces well-behaved, more predictable behavior in a rather surprising and unintuitive way.

Consider a grayscale picture g, analogous to a matrix of individual pixel intensities g_j. Now, let us take only the magnitudes of the Discrete Fourier Transform (DFT) G of this image, given by

$$|G[k]| = \left|\sum_{j=0}^{n-1} g_j e^{-2i\pi \frac{k}{n} j}\right|.$$

Here, n represents the number of pixels in g and k is a frequency index.

The DFT decomposes an image into frequency components; typically, if one knows the DFT of an image, the original image can be retrieved just by using the inverse of the DFT. However, taking the absolute value as we did above removes information on the phase: the phase determines where structures appear in the image, while the magnitude determines their strength. This leads to the (standard) phase problem, where we ask: when is it possible to recover a signal from its Fourier (DFT) magnitudes alone? A simplified example of the phase problem is shown in Figure 1. In a sense, if we can recover the DFT of the image, phase and all, then we can recover the image itself.

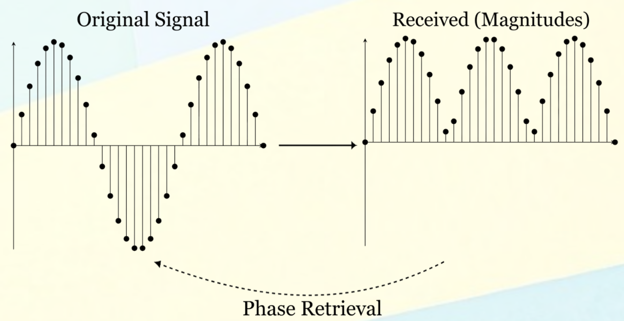

Figure 1: *Simple diagram of phase retrieval in 1D. Image created by Aldas Kriauciunas.*

A Brief Physical Motivation

The need for a solution to the standard phase problem rapidly expanded in the late 19th and early 20th century with the invention of X-ray crystallography: the study of molecular structures through the diffraction of X-rays in crystals (which has led to a great many discoveries, such as the structure of DNA; see Figure 2). Typically, one is only able to detect the intensity of light, not the phase, despite both being needed to reconstruct a complete map of the structure being investigated. Although we used the example of crystallography, phase retrieval is a fundamental problem in broader imaging and signal processing due to quantum measurement uncertainty; specifically, it is impossible to know the amplitude and phase of any one particle perfectly, which naturally produces the phase problem in fundamental physics.

Formalizing the Problem

The standard phase problem can be stated in more generality, incorporating the cases when, e.g., the received information is not perfectly the DFT of the original image we are attempting to reconstruct. Be-

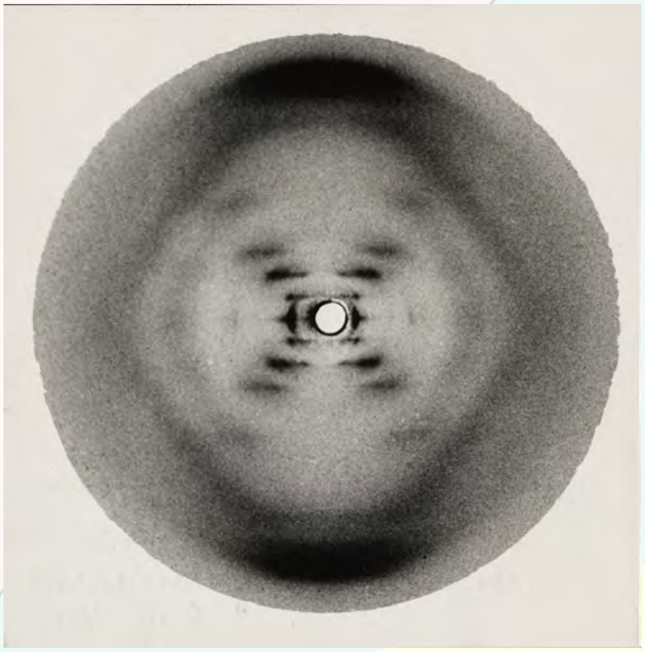

Figure 2: *Rosalind Franklin's famous x-ray crystallographic map of DNA. [1]*

fore stating the problem, we first define the inner product (dot product) of the n-dimensional complex vectors $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ to be

$$\langle x, y\rangle = \sum_{i=1}^{n} x_i \bar{y}_i = x_1\bar{y}_1 + x_2\bar{y}_2 + \cdots + x_n\bar{y}_n$$

with the norm of any vector defined as $\|x\| = \sqrt{\langle x, x\rangle}$.

The generalized phase problem is stated as follows:

Generalized Phase Retrieval Problem (GPR): Let $x = (x_1, x_2, \ldots, x_n) \in \mathbb{C}^n$ be an unknown signal and let $a_k \in \mathbb{C}^n$ for $k = 1, \ldots, m$ be sensing vectors. Given a measurement vector with elements $y_k = |\langle a_k, x\rangle|$, is it possible to recover x up to global phase factor $e^{i\theta}$, $\theta \in [0, 2\pi)$?

It is worth noting that taking $m = n$ and

$$a_k = \left\{e^{-2\pi ikj/n}\right\}_{j=0}^{n-1} \text{ (the } k\text{-th Fourier basis vector)}$$

returns the standard phase problem (to see this, notice that $\langle a_k, g\rangle = G[k]$). But what if each a_k is not restricted to just be from the Fourier basis? What if each element is a small perturbation of the basis, or something entirely non-deterministic? To investigate these questions, we look to the objective function:

$$f(z) = \frac{1}{2m}\sum_{k=1}^{m}\left(|y_k|^2 - |\langle a_k, z\rangle|^2\right)^2. \qquad (1)$$

To solve the GPR, we want to find for what z does $f(z)$ obtain its (absolute) minimum value. In lax terms, we want to find the z closest to the original set x (our matrix we want to recover) using just the inner products with each a_k (the measurements).

We want to optimize f. However, f is nonconvex (and so, f has multiple local minima), so there is no obvious reason to expect that, in whatever optimization routine we employ, we would obtain global optimization from an arbitrary starting guess. In fact, for most deterministic sets $\{a_k\}$, there is not much else to do but hope that whatever optimization routine being employed will actually work.

Now, we do something somewhat counterintuitive: we let each a_k be chosen randomly. Explicitly, we choose each a_k from a complex Gaussian:

$$a_k = \frac{1}{\sqrt{2}}(X_k + iY_k)$$

where X_k, Y_k are vectors taken from an n-dimensional Gaussian (normal) distribution. Here, each a_k is chosen from the same distribution and without dependence on other a_i's for $i \neq k$ (this is called the independently and identically distributed (i.i.d.) assumption). Now, we let the number of measurements increase. It was shown by Sun, Qu and Wright [3] that the expectation of the objective function for the GPR actually obtains a well-behaved, convex geometry (convexity is desirable because any local minimum is automatically a global minimum), which can be solved by standard optimization methods with high probability! In other words, the graph of f is convex around the solutions, and so many standard algorithms for optimizing f are almost always able to solve the GPR.

To see why this happens, we do not need to look any further than the expectation (mean). We denote $\mathbb{E}_N[f(z)]$ as the expectation when the a_k are drawn from the complex Gaussian distribution. Explicitly, as proved in [3], this expectation is as follows:

$$\mathbb{E}_N[f(z)] = \|z\|^4 + \|x\|^4 - \|z\|^2\|x\|^2 - |\langle x, z\rangle|^2.$$

Compare $\mathbb{E}_N[f]$ to the equation (1): in the random (Gaussian) sampling regime, we suddenly have only a dependence on norms and inner products: morally, everything that happens in the complex plane $\mathbb{C}$ happens in arbitrary dimension $\mathbb{C}^n$, as $\mathbb{E}_N[f]$ relies only on radial distance and signal alignments! One can show that the only z which minimize $\mathbb{E}_N[f(z)]$ are exactly $z = xe^{i\theta}$; these are the global minima which solve the GPR (see Figure 3).

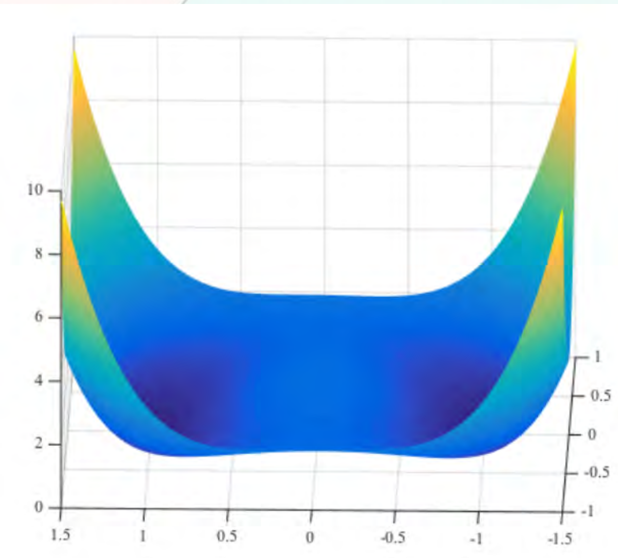

Figure 3: *Approximate function landscape of $\mathbb{E}_N[f]$. [3] Notice the highly regular, convex geometry on the right and left hand sides individually.*

Curiously, then, by having random and numerous enough a_k 's, the GPR becomes essentially uniquely solvable, contrary to the deterministic version of the problem.

But what if this is an isolated event? Perhaps this is just a nice feature of the Gaussian distributions? Let us look at another distribution, this time defined on $\mathbb{C}$ based on our prior observations. Choose ϕ_k i.i.d. uniformly on $[0, 2\pi)$. Then, map $\phi_k \mapsto e^{i\phi_k} = a_k$ for all k. That is, we choose each a_k to be a random point on the complex unit circle. We will call this distribution $\mathbb{T}_d$. Finding the expected value, it turns out that:

$$\mathbb{E}_{\mathbb{T}_d}[f(z)] = \mathbb{E}_N[f(z)] - \frac{1}{2}\sum_{i=1}^{n}\left(|z_i|^2 - |x_i|^2\right)^2 .$$

Now, we have a greater dependence on the individual alignment of the *elements* of z and x. Indeed, this additional term makes it more probable that $f(z)$ retains some nonconvexity, which is unfavorable for optimization. Despite this, it can be shown that, in high dimension (large n), $\mathbb{E}_{\mathbb{T}_d}[f]$ almost always obtains a global convex geometry! We are again able to solve the GPR with a vast range of optimization algorithms with relative ease.

Where does this fail, though? What if we want to test a distribution less "regular"? In particular, Gaussian and uniform distributions have highly desirable properties such as radial symmetry or invariance to certain transformations, so what exactly about a distribution makes it well-suited to solve the GPR? To see what happens, we must create a less symmetric distribution.

Building and Testing Intuition

Let us look at a low-outcome discrete distribution to push the limitations of our theory: choose each $a_k \in \mathbb{C}$ i.i.d. from the set $D = \left\{1, e^{i\pi/4}, i\right\}$ with equal probability for each element. Unlike the Gaussian or complex unit circle distributions, the mean value of the elements in D is nonzero, and we also lose radial symmetry, leading to far fewer cancellations when computing $\mathbb{E}_D[f]$ directly[1].

Before jumping straight into computations, we should make a hypothesis. Based on the discussion up to this point, it is reasonable to intuit that we should be able to reconstruct an image with ease when a_k is taken from $\mathbb{T}_d$, with more difficulty when a_k is taken from D, and with the greatest difficulty when a_k is the deterministic Fourier basis.

We return now to our grayscale image and check if our intuition can be validated computationally. For our grayscale image, we will start with a microscopic image of an aspirin crystal and then take only its Fourier magnitudes (i.e. removing all phase information). We apply a Gerchberg-Saxton type algorithm (see [2]) choosing a_k from either $\mathbb{T}_d$, D, or the standard Fourier basis (we will omit the complex Gaussian a_k for space). The results of our optimization of f versus the original image we hoped to reconstruct is shown in Figure 4.

[1] To be exact, the formula $\mathbb{E}_D[f]$ has exactly $n!/(n-4)! + 6n!/(n-3)! + 4n(n-1)$ *more uncanceled cross terms* than $\mathbb{E}_N[f]$ hence why the full expression is omitted.

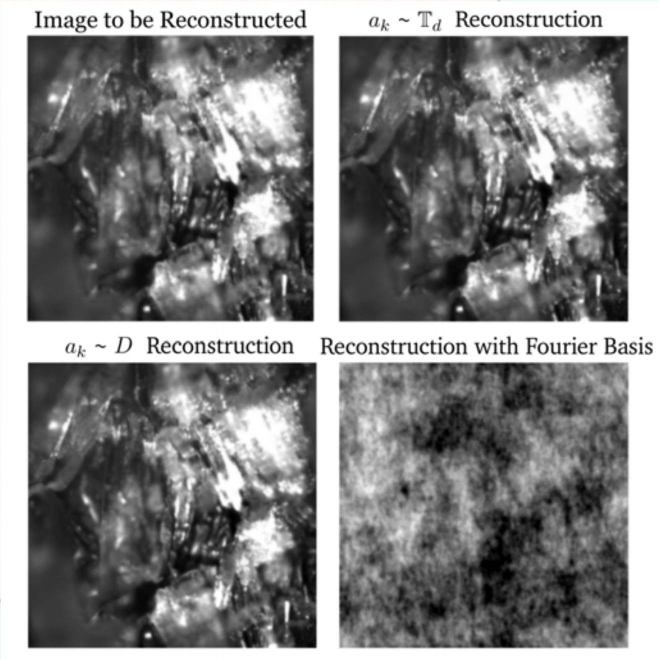

Figure 4: *Original aspirin crystal image (top left) versus reconstructions for different choices of a_k.*

Interestingly, our intuition is correct! In fact, what we find is even stronger when looking at the error of the reconstruction at each iteration of the algorithm, shown in Figure 5.

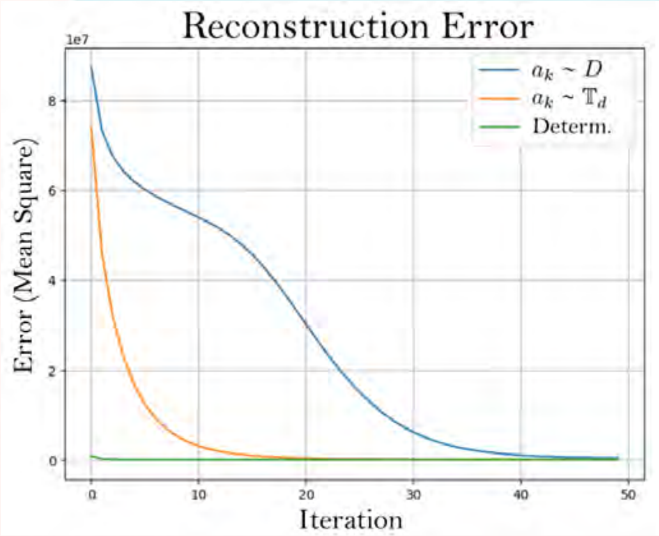

Figure 5: *Average error of image reconstruction at each iteration of the Gerchberg-Saxton type algorithm (up to 50 iterations).*

There are three crucial observations to make based on the error trends:

1. The error for the deterministic (Fourier) basis almost immediately shoots to zero, despite the reconstructed image being extremely poor. This is exactly the algorithm getting trapped in the local minima of the nonconvex f.

2. When $a_k \sim D$, it takes far more iterations for convergence compared to $a_k \sim \mathbb{T}_d$; however, unlike the deterministic case, the image is successfully reconstructed, hinting at an "incomplete" convexity.

3. For $a_k \sim \mathbb{T}_d$, we get rapid convergence to the target set, as expected from the nice properties of $\mathbb{E}_{\mathbb{T}_d}[f]$ we have discussed.

All three of these observations directly correlate with our original intuition-based hypothesis! By inserting randomness into our problem, we have indeed made it easier to predict the behavior of the solutions through geometric characterization.

Random measurements effectively isotropize the problem: they collapse high-dimensional complexity into a 2D geometry governed entirely by alignment with the signal. In this way, randomness transforms a difficult nonconvex optimization problem into one with predictable geometric behavior. Almost paradoxically, to become more certain, it took a bit of uncertainty.

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From Integer Partitions to Young Tableaux

By Preetinder Kaur

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In combinatorics, one often asks: "How many ways are there ...?" In this article, we will explore one such question for different mathematical structures.

Have you ever considered how many ways there are to write a given positive integer n as a sum of positive integers?

Example Let's do a few examples.

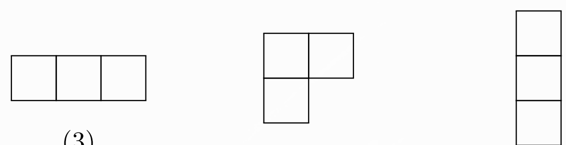
n	**Partitions**	**Total** $p(n)$
1	1	1
2	2, 1+1	2
3	3, 2+1, 1+1+1	3
4	4, 3+1, 2+2, 2+1+1, 1+1+1+1	5
5	5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, 1+1+1+1+1	7

Table 1: *Integer Partitions for* $n = 1$ *to* 5

Note that we consider $2+1$ and $1+2$ to be the same; for convenience, we always write summands in decreasing order.

We can represent these decompositions more compactly by listing just the summands in decreasing order. For example, we write $(2,1)$ for $2+1$ and $(1,1,1)$ for $1+1+1$. In mathematics, such a decreasing sequence of positive integers $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_d)$ that sum to n is called a *partition* of n, and we denote this by $\lambda \vdash n$. Then the question becomes: How many partitions are there for a given positive integer n? As simple as the question sounds, there is still no known closed formula for the number of partitions of a given integer (see Lectures on Integer Partitions by Herbert S. Wilf [4]).

Let's change our perspective a little and try to answer some different questions related to partitions. It turns out that integer partitions can be visualized using boxes! Given a partition $\lambda \vdash n$, we arrange n boxes so that the i-th row contains exactly λ_i many boxes for all $1 \leq i \leq d$. The resulting shape is called the *Young diagram* of shape λ. For example, the Young diagrams corresponding to the partitions (3), $(2,1)$, and $(1,1,1)$ of 3 are shown below, respectively.



(3) $(2,1)$ $(1,1,1)$

Figure 1: *Young diagrams corresponding to partitions of* 3

Young diagrams give us a visual way to think about partitions. But we can do something even more interesting: we can fill these boxes with numbers! However, it is important to set some filling rules.

Counting Young Tableaux

A *Young tableau* is a Young diagram filled with the integers $1, 2, \ldots, n$, each appearing exactly once.

Example For $\lambda_1 = (3,2) \vdash 5$, examples of tableaux of shape λ_1 are:

3	5	2
1	4	

5	2	4
1	3	

4	2	1
3	5	

Young tableaux of shape $(3,2)$

For $\lambda_2 = (2,2,1) \vdash 5$, examples of tableaux of shape λ_2 are:

$$\begin{array}{|c|c|}\hline 3 & 5 \\ \hline 1 & 2 \\ \hline 4 \\ \cline{1-1} \end{array} \qquad \begin{array}{|c|c|}\hline 5 & 4 \\ \hline 3 & 1 \\ \hline 2 \\ \cline{1-1} \end{array} \qquad \begin{array}{|c|c|}\hline 2 & 5 \\ \hline 4 & 3 \\ \hline 1 \\ \cline{1-1} \end{array}$$

Young tableaux of shape $(2, 2, 1)$

Now, let's ask: How many such tableaux exist for a given shape? To count the number of Young tableaux for a fixed shape λ with n boxes, we simply treat the diagram as a set of n distinct placeholders — since there are no restrictions on where numbers go, the shape plays no role at this stage. The only rule we need to pay attention is that each integer from $\{1, 2, \ldots, n\}$ must appear exactly once, so we have n options for the first box, $(n-1)$ options for the second, and so on, until only one number remains for the final box. Therefore, the total number of ways to fill any Young diagram of size n is simply $n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1 = n!$.

Counting Standard Young Tableaux

What happens if we require the entries to be placed in a way that they increase strictly across rows and down columns? Such a tableau is called a *standard Young tableau*.

Example For $\lambda_1 = (3, 2) \vdash 5$, examples of standard Young tableaux of shape λ are:

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 \\ \cline{1-2} \end{array} \qquad \begin{array}{|c|c|c|}\hline 1 & 3 & 5 \\ \hline 2 & 4 \\ \cline{1-2} \end{array} \qquad \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & 5 \\ \cline{1-2} \end{array}$$

Standard Young tableaux of shape (3,2)

For $\lambda_2 = (2, 2, 1) \vdash 5$, examples of standard Young tableaux of shape λ are:

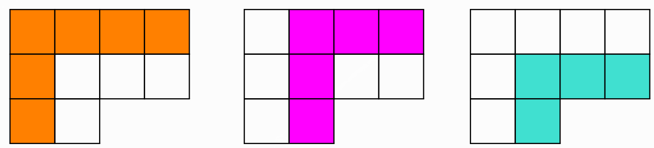
$$\begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 \\ \cline{1-1} \end{array} \qquad \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 \\ \cline{1-1} \end{array} \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 \\ \cline{1-1} \end{array}$$

Standard Young tableaux of shape (2, 2, 1)

Now, let's ask again: How many such tableaux exist for a given shape λ? Let's denote this number by f^λ. Well, now the answer is not that simple because of all those restrictions we added. One may think that with all these restrictions we have less possibilities, and hence counting would be easier. However, the issue is not counting but formulating the pattern we have for a fixed shape. For this counting problem, we need another tool known as a hook.

Definition For a Young diagram λ, each box determines a *hook*, which consists of that box and all boxes in its row to the right of the box and in its column below the box. Moreover, the *hook length* of a box is the number of boxes in its hook and is denoted by $h(i, j)$ for the box in position (i, j), i.e. row i and column j, of λ.

Example Consider the shape $\lambda = (4, 4, 2) \vdash 10$. The hook of the box $(1, 1)$ is given in Figure 2a, and its hook length is $h(1, 1) = 6$. The hook of the box $(1, 2)$ is given in Figure 2b, and the hook length is $h(1, 2) = 5$. The hook of the box $(2, 2)$ is given in Figure 2c, and the hook length is $h(2, 2) = 4$.



(a) **(b)** **(c)**

Figure 2: *Some hooks in shape* $\lambda = (4, 4, 2)$

Example Let $\lambda = (3, 1) \vdash 4$. We want to count how many standard Young tableaux of this shape exist. Rather than guessing a formula, let us just list them all directly:

$$T_1 = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1} \end{array} \quad T_2 = \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1} \end{array} \quad T_3 = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1} \end{array}$$

So there are 3 such tableaux, i.e. $f^{(3,1)} = 3$. Now let us understand why this is 3 in a way that generalizes.

Think about the way you start placing the numbers: one can either start placing from the smallest number or the biggest number. Here we track the largest number. Observe that there are special spots where we can place the largest number 4. In any standard Young tableau, the largest entry must go in a *corner*, that is, a box with nothing to its right and nothing below it. If it were anywhere else, there would be a neighbor that would have to be even larger, which is impossible. For $\lambda = (3, 1)$, the corners are boxes $(1, 3)$ and $(2, 1)$. So, the entry 4 must go in one of them. We can see this in our list: in T_1 the entry 4 sits in corner $(2,1)$, and in T_2 and T_3 it sits in the corner $(1,3)$.

This gives us a natural way to split the count. Once we decide where 4 goes, we remove that corner and fill the remaining shape with $1,2,3$ as a standard Young tableau. So we are left with two different shapes: shape $(2,1)$ and shape (3), giving us

$$f^{(3,1)} = f^{(2,1)} + f^{(3)},$$

where $f^{(2,1)}$ counts the tableaux of the shape left after removing corner $(1,3)$, and $f^{(3)}$ counts the tableaux left after removing corner $(2,1)$. We can verify these directly:

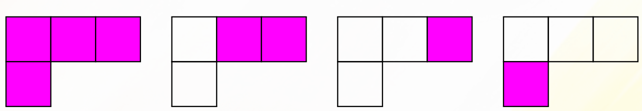
$$\text{Shape } (2,1): \quad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array} \quad \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array} \quad \Rightarrow \qquad f^{(2,1)} = 2,$$

$$\text{Shape } (3): \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline\end{array} \quad \Rightarrow \qquad f^{(3)} = 1.$$

So $f^{(3,1)} = 2 + 1 = 3$, which matches.

The same argument applies recursively to each smaller shape, and eventually every shape reduces to a single box with exactly one filling.

Now, let's discuss how this number can be expressed in terms of hook lengths. Recall that the hook length $h(i,j)$ counts how many boxes are in the hook of the box at position (i,j).



Hook of box $(1,1)$ *Hook of box $(1,2)$* *Hook of box $(1,3)$* *Hook of box $(2,1)$*

Thus, for $\lambda = (3,1)$, the hook length of each box in the Young diagram of shape $(3,1)$ is shown below:

$$\begin{array}{|c|c|c|}\hline 4 & 2 & 1 \\ \hline 1 & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}$$

Interestingly, if we divide the number of Young tableaux, $n! = 4! = 24$, by the product of all hook lengths $4 \cdot 2 \cdot 1 \cdot 1 = 8$, we also get 3, which is the number of standard Young tableaux of shape $\lambda = (3,1)$. Is this a coincidence?

In fact, this is no coincidence! For any shape $\lambda \vdash n$,

$$f^{\lambda} = \frac{n!}{\prod_{(i,j)\in\lambda} h(i,j)}.$$

This formula is known as the *Hook Length Formula* [2]. Since this was first proved by Frame, Robinson, and Thrall in [1], many different proofs have been given. It is natural to ask *why* the hook lengths appear in this formula at all. Greene, Nijenhuis, and Wilf in [3] answered this by reinterpreting the recursion probabilistically. Rather than directly counting tableaux, they construct a random walk on the diagram and prove that the probability of terminating at each corner box is exactly $f^{\lambda\setminus\text{corner}}/f^{\lambda}$, where $f^{\lambda\setminus\text{corner}}$ denotes the number of standard Young tableaux of the shape with the corner removed. This implies the recursive relation

$$f^{\lambda} = \sum_{\text{corners}} f^{\lambda\setminus\text{corner}}.$$

The walk works as follows: pick one of the n boxes uniformly at random, i.e. with probability $1/n$, as your starting box. From your current box (i,j), look at its hook and move to one of those remaining boxes in its hook chosen uniformly at random, with probability $\dfrac{1}{h(i,j)-1}$. Repeat this process until you reach a corner box, where the walk terminates.

Let us do this for the shape $\lambda = (3,1)$. The two corners are the boxes in the $(1,3)$ and $(2,1)$ positions. Suppose the walk starts at box $(1,1)$, whose hook covers the entire diagram. From $(1,1)$ we move to one of the remaining three hook boxes $(1,2)$, $(1,3)$, or $(2,1)$ with equal probability $1/3$. If we land on $(1,3)$ or $(2,1)$, we are already at a corner and stop. If we land on $(1,2)$, its hook contains only $(1,3)$, so we are forced to move there with probability 1 and stop. So from $(1,1)$, the walk always terminates at a corner, just through different routes as shown in Figure 3.
Greene, Nijenhuis, and Wilf proved that when we average over all possible starting boxes and all possible random walks, the probability of terminating at corner $(1,3)$ is $f^{(2,1)}/f^{(3,1)} = 2/3$, and the probability of terminating at corner $(2,1)$ is $f^{(3)}/f^{(3,1)} = 1/3$. Since every walk must terminate somewhere, these probabilities sum to 1, which is precisely the recursion in our example $f^{(3,1)} = f^{(2,1)} + f^{(3)}$. By induction on the shape λ, this argument can be generalized to any shape and gives the Hook Length formula.

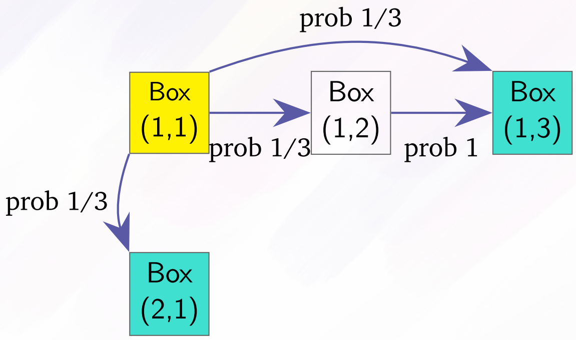


Figure 3: *Walk starting at box* $(1,1)$

Counting Standard Perfect Tableaux

Now, let's focus on a special case of standard Young tableaux of shape $\lambda = (n, n, \ldots, n)$ with n components, which is just an $n \times n$ square, and examine its enumeration.

Definition A standard Young tableau of shape $\lambda = (n, n, \ldots, n)$ where the labels for the main diagonal, from top-left to bottom-right, are fixed to be perfect squares is referred to as a *standard perfect tableau of shape* λ or *standard perfect tableau of size* n. The collection of such tableaux is denoted by SPT_n.

Note that in a standard perfect tableau of shape λ, for all $k \leq n$, the label for box (k,k) is always k^2.

Example For $\lambda = (4,4,4,4) \vdash 16$, corresponding examples of standard Young tableau and standard perfect tableau are given below:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Standard Young Tableau of shape λ

1	2	5	10
3	**4**	6	11
7	8	**9**	12
13	14	15	**16**

Standard Perfect Tableau of shape λ

Let $\nu = (n-1, n-1, \ldots, n-1)$ be the $(n-1)\times(n-1)$ square shape. Then the *skew shape* λ/ν, consisting of the boxes in the nth row and the nth column of λ, is called the *nth reflected hook* of a tableau in SPT_n.

Observe that, due to the structure of a standard perfect tableau of shape λ, the numbers used to fill the boxes of shape ν can never appear in the nth reflected hook.

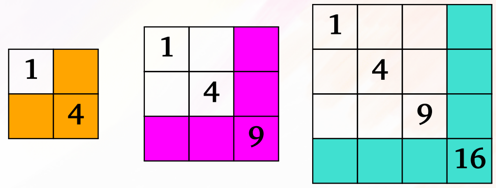


Figure 4: *Reflected hooks for Standard Perfect Tableau of size* $2, 3, 4$*, respectively.*

From now on, the numbers that can be used to fill the boxes of the tableau while satisfying the necessary conditions are referred to as *admissible labels*. The cardinality of the set of all admissible labels of the nth reflected hook is denoted by h_n. Note that while the nth reflected hook includes the corner box (n,n), the label n^2 placed there is fixed and hence is not considered admissible.

Lemma 1. *The set of all admissible labels for the nth reflected hook is given by*

$$\{(n-1)^2+1, (n-1)^2+2, \ldots, n^2-1\},$$

and hence $h_n = 2(n-1)$.

Proof. It is known that the labels of a standard Young tableau of shape λ are $1, 2, \ldots, n^2$. Since each row and column must be increasing in a standard perfect tableau, the labels less than $(n-1)^2$ can only appear above or to the left of the $(n-1, n-1)$th box. Hence, all labels from $\{1, 2, \ldots, (n-1)^2\}$ are contained in the top-left $(n-1)\times(n-1)$ subregion of shape ν. The remaining labels are $(n-1)^2+1, \ldots, n^2$, and the position of the label n^2 is always fixed. Therefore, the set of all admissible labels for the nth reflected hook is $\{(n-1)^2+1, (n-1)^2+2, \ldots, n^2-1\}$, and consequently, $h_n = (n^2-1) - \big((n-1)^2+1\big) + 1 = 2(n-1)$. $\square$

Example Consider the standard perfect tableau from the previous example, and observe that no numbers from $\{1, 2, 3, \ldots, 9\}$ can leave the top-left 3×3 square. This follows from the rules of a standard Young tableau; no entry greater than 9 can appear above or to the left of the box containing 9. Consequently, all entries larger than 9 must be placed to the right or below this position.

In the given tableau below (see Figure 5), the third reflected hook of a tableau in SPT_3 is highlighted in yellow, and admissible labels are $\{5, 6, 7, 8\}$. Then, $h_3 = 2 \cdot (3-1) = 4$. Note that 9 is a perfect square, so it is fixed on the main diagonal and hence not included in the reflected hook and h_3 count.

1	$2/3$	$*$
$3/2$	**4**	$*$
$*$	$*$	**9**

Figure 5: *Illustration of all possible standard perfect tableaux of size* 3.

A filling of the nth reflected hook that satisfies the rules of a standard Young tableau is called an *admissible filling*, and the total number of such fillings is denoted by $\mathcal{H}_n$.

Lemma 2. *In the nth reflected hook of any tableau in* SPT_n*, the filling of the nth column uniquely determines the filling of the nth row, and vice versa.*

Proof. If the column of the nth reflected hook in a tableau from SPT_n is filled using admissible labels, then the corresponding row must be filled with the remaining labels in increasing order, as required by the standard Young tableau conditions. Since there is only one increasing arrangement of the remaining admissible labels, filling the column uniquely determines the row, and vice versa. Hence, filling either the row or the column completely determines the nth reflected hook of the tableau. □

Lemma 3. *The values $\mathcal{H}_n$ are enumerated by the Central Binomial Coefficients. That is,*

$$\mathcal{H}_n = \binom{2(n-1)}{n-1} = n \cdot C_{n-1}$$

where C_n is the n-th Catalan number. These numbers are also known as the Type B Catalan Numbers. This sequence appears as A000984 in the OEIS.

Proof. By the definition of the standard perfect tableau and Lemma 2, there are only $n-1$ boxes to be filled. By Lemma 1 the number of admissible labels for these boxes is $h_n = 2(n-1)$. Thus,

$$\mathcal{H}_n = \binom{\text{Number of admissible labels}}{\text{Number of available boxes}} = \binom{h_n}{n-1} = \binom{2(n-1)}{n-1}$$

□

Table 2 records the number of admissible fillings for given standard Young tableaux of sizes up to 5, using Lemma 3.

n	h_n	$\mathcal{H}_n$
1	0	1
2	2	2
3	4	6
4	6	20
5	8	70

Table 2: *Number of admissible labels and fillings for* $1 \leq n \leq 5$.

Example For a tableau in SPT_4, the set of all admissible labels for the 4th reflected hook is $\{10, 11, 12, 13, 14, 15\}$. Thus, $h_n = 2(n-1) = 2(4-1) = 2(3) = 6$. We compute $\mathcal{H}_4$ (see Figure 6). Using Lemma 2, we focus on the 4th row, highlighted in pink. Note that the last box in the nth row, highlighted in teal, lies on the main diagonal and contains the fixed label 16. In the 4th row, there are $n-1 = 4-1 = 3$ boxes to be filled, and the number of admissible labels is $h_4 = 6$. Therefore,

$$\mathcal{H}_4 = \binom{h_4}{4-1} = \binom{6}{3} = 20.$$

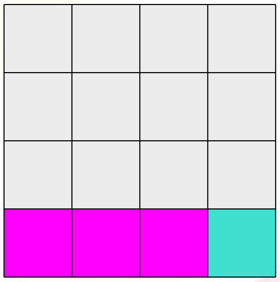


Figure 6: *The 4th reflected hook of* SPT_4*, with the $n-1=3$ fillable boxes highlighted in pink and the fixed diagonal entry in blue.*

Lemma 4. *The sizes of* SPT_n *are enumerated by the geometric product of the Central Binomial Coefficients. That is,*

$$|\,\mathrm{SPT}_n\,| = \prod_{i=1}^{n} \mathcal{H}_i.$$

This sequence appears as A007685 in the OEIS.

Proof. Using Lemma 1, we know that admissible labels for any tableau in SPT_n can only contain values from $(n-1)^2+1$ to n^2-1. Furthermore, for any tableau in SPT_m with $m \geq n$, the top-left $(n-1) \times (n-1)$ region always contains the fillings of a standard perfect tableau of size $n-1$. It follows that the fillings of each reflected hook are independent of the fillings of all other reflected hooks, as illustrated in Figure 7. So, to get $|\,\mathrm{SPT}_n\,|$, we multiply all the $\mathcal{H}_i$ for all $i \in \mathbb{N}$ such that $1 \leq i \leq n$. This yields

$$|\,\mathrm{SPT}_n\,| = \mathcal{H}_1 \times \mathcal{H}_2 \times \ldots \times \mathcal{H}_{n-1} \times \mathcal{H}_n = \prod_{i=1}^{n} \mathcal{H}_i.$$

$\square$

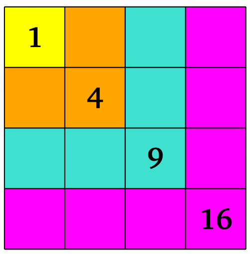


Figure 7: *nth Reflected hook for a tableau of shape λ, for* $1 \leq n \leq 4$

Acknowledgment

This article is the first part of an undergraduate research project conducted in 2024–2025 under the supervision of Dr. Özlem Uğurlu.

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Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called "EvenQuads" to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.

Verdiana Masanja (b. 1954)

Masanja was educated in her native Tanzania and in Germany, and is the first Tanzanian woman to earn a doctorate in mathematics. She is now professor of computational mathematics at The Nelson Mandela African Institution of Science and Technology in Tanzania. Masanja's research is in fluid dynamics and mathematical modeling, with publications related to water pollution, food security, and disease spread. She has also published both a textbook for teacher trainees and a university number theory textbook. Masanja has supervised 7 doctorates so far. She received the University of Dar es Salaam Golden Outstanding Award (2011) for her contributions to the teaching and learning of mathematics.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.

Mary Lucy Cartwright (1900–1998)

Mary Cartwright earned her DPhil from Oxford University (1930). Her more than 90 papers made important contributions to the theory of functions and differential equations, and a collaboration on radar analysis laid the foundation for chaos theory. Cartwright was director of studies in mathematics at Girton College (Cambridge) 1936–1949 and then Mistress of Girton. She was promoted to Reader (1959) and was President of the LMS (1961–1963). Cartwright was the first woman mathematician elected to Fellow of the Royal Society (1947) and the first woman to receive the Royal Society's Sylvester Medal (1964) and the LMS De Morgan Medal (1968). Cartwright was elevated to Dame Commander of the British Empire in 1969.

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Finding Your Place in the Mathematical Community

By Sophia Huang[1] and Preetinder Kaur[2]

[1] Undergraduate Student, Department of Electrical and Computer Engineering, Saint Louis University
[2] Undergraduate Student, Department of Mathematics and Statistics, Saint Louis University

Preetinder Kaur, Sophia Huang, and Samantha Doll (from left to right)

A Memorable First Conference Experience — *Sophia Huang*

My experience at the 2026 Nebraska Conference for Undergraduate Wisdom in Mathematics (NCUWM) conference has been one of continuous learning and a memorable introduction into mathematics. The sessions presented by fellow undergraduates provided broad exposure to numerous fields within mathematics, at a depth that was both digestible and appropriate at the undergraduate level. Attending these short talks made the possibilities of mathematical research abundantly clear, in which topics ranged from modeling the spread of disease to the investigation of Costello multiplication, a fictional proof used in a classic comedic sketch. Spread throughout the conference schedule were panel discussions hosted by experts in industry and academia, in which the panelists offered a variety of personal and professional advice. The stories shared in these conversations were meaningful in that they revealed prospective career pathways but also demonstrated the results of persistent effort and tireless dedication. It was especially motivating to see representation in mathematics, as many of the speakers were once the only women in their graduating class or company department. The community and sincere interest from the conference has made a lasting impression that has impacted my perspective on mathematics and on learning as a whole.

From Attendee to Presenter — *Preet Kaur*

Returning to the 28th Annual Nebraska Conference for Undergraduate Wisdom in Mathematics (NCUWM) this January felt like a full-circle moment. Last year, I was there to observe, but this year I participated as a presenter. There is a specific kind of shift that happens when you see your name in the program; standing at the front of the room for my first conference talk was both exciting and slightly surreal. Realizing that peers and mentors had specifically chosen to listen to my work made the research feel “real” in a way it never had in a lab or a classroom.

I presented my talk, “Binary Pullback Parking Functions,” based on research I conducted during the NSF Research Experience for Undergraduates (REU) “Combinatorics and Coding Theory in the Tropics” program at the University of Puerto Rico in Ponce in summer 2025. Sharing work that began months earlier in a completely different setting added another layer of meaning to the experience. What started as an intense summer research project had now grown into something I could communicate to a broader audience.

What makes NCUWM so unique is the sheer breadth of ideas it brings together. Throughout the weekend, I attended presentations that connected mathematics to biology, cryptography, games, and even everyday decision-making. These sessions were a needed reminder that math isn’t a rigid, narrow path; it’s

Preetinder Kaur presenting at NCUWM 2026

a versatile tool that adapts to almost any field you're passionate about. This sense of possibility was reinforced by the keynote speakers, who were refreshingly honest about their own career detours and moments of self-doubt. They made success feel like something attainable and human rather than a distant, perfect standard.

The most motivating part of the weekend, however, was the environment. It is rare to be in a space with hundreds of people who are just as invested in mathematics as you are. The atmosphere was entirely collaborative, with students cheering each other on and asking supportive questions.

I am deeply grateful to Dr. Dorsa Ghoreishi for the funding and for traveling with us to the conference. I also want to thank both her and Dr. Özlem Uğurlu for their guidance in helping me prepare the presentation; their support turned a daunting task into a confident milestone. Attending NCUWM this year marked a clear transition for me, from observing the mathematical community to actively contributing to it. Experiences like this remind me that growth often happens when we step slightly beyond our comfort zones and allow ourselves to be seen, heard, and challenged in new ways.

Women Mathematicians Corner

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Deborah Lowenberg Ball (b. 1950s)

Deborah Loewenberg Ball earned her PhD in Math Education and Policy from Michigan State University (1988). She holds two named professorships in the School of Education and is the founding Director of TeachingWorks (2011), all at the University of Michigan. Ball has over 150 publications related to teacher preparation. Her research has been supported by more than 20 NSF grants. Ball was honored with the AWM Hay Award (2009), the AACTE Pomeroy Award (2014), and the IMU Felix Klein Medal (2017). She was elected to the NAE (2007) and the American Academy of Arts and Sciences (2014), and is a Fellow of both the AMS (2013) and the AERA (2013). Ball was appointed to the NSB (2013–18) by President Obama.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

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EvenQuads

YoungJu Choie (b. 1959)

YoungJu Choie completed her PhD in mathematics at Temple University (1986) and is currently a professor at Pohang University of Science and Technology in South Korea. She studies number theory and modular forms and has published over one hundred research articles, as well as two recent books. Choie is an editor of the Springer *Journal for Research in Number Theory*. She served as president of the Korean Women in the Mathematical Sciences (2017). She was named "best woman scientist of the year" by the Korean Ministry of Science and Technology (2005), is an AMS Fellow (2013), and received the Science and Technology Medal of Innovation Medal from the President of South Korea (2022).

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

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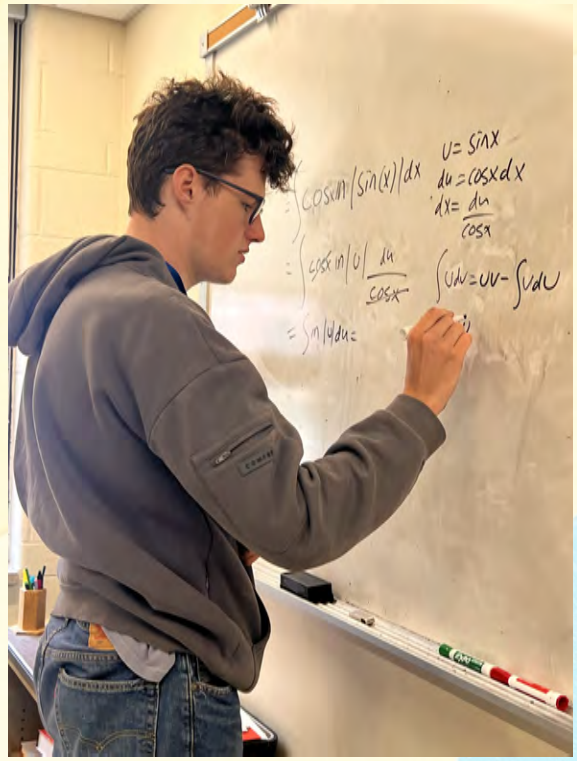
Math in the Wild Challenge

Caleb Kernaghan — Studying at Help Sessions, Department of Mathematics and Statistics, Saint Louis University

Andrew Moritz, Richard Niemczura, and Taylor Pike — Math being discussed and thought of, in the wild, on the STEM Wellness Walks



Stevie Potter — Working through an integral at Help Sessions, Department of Mathematics and Statistics, Saint Louis University

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Submit your photo to be featured in the next issue!

Cryptic Crossword Puzzle

By Nathaniel Alloway[1] and Evan Socher[2]

[1] Master's Student, Department of Mathematics and Statistics, Saint Louis University
[2] Undergraduate Student, Department of Mathematics and Statistics, Saint Louis University

Puzzle Instructions

A standard cryptic clue has two main parts, the definition and the wordplay. The clue's definition will almost always be found either at the beginning or the end of a clue. The definition can be a dictionary definition, a playful twist on words, or a 'definition by example,' just like in a standard crossword puzzle.

What distinguishes these clues from those found in a standard crossword puzzle is the other half: the wordplay. Wordplay can be broken down into "fodder" and "indicators." The fodder are the words or letters we are manipulating, and the indicators tell us how we manipulate them. Often, identifying which parts of a clue are definition, fodder, and indicators is half the battle! The length of the answer is represented by the number in the ending parentheses, and if an answer has multiple words, it is represented as $(n_1, \dots, n_k)$ where n_i is the length of the i-th word.

Example 1 A horizontal arrangement grows without bounds (3)

$$\begin{aligned} \text{Definition} &= \text{A horizontal arrangement} \\ \text{Fodder} &= \text{grows} \\ \text{Indicator (deletion)} &= \text{without bounds} \end{aligned}$$

Here, we see (3), implying our answer will be one 3-letter word. If we look at the word "grows" and take it "without bounds," removing the so-called bounding letters **g** and **s**, then this gives us the word "row," which is "a horizontal arrangement."

In general, the clue's punctuation can be ignored and may be used to make the clue read naturally or even mislead the solver. An exception to this is the use of a question mark, which typically indicates that the definition is a bit of a stretch, though not always. Additionally, a clue may also have connective words between the wordplay and definition like "is," "are," "for," or "gives." Aside from these, each word in the clue will be relevant to the structure of the clue and, with the exception of certain special kinds of clue, there will be no overlap.* This means that if a word is used as, say, part of a definition, then it will not also be part of the fodder or indicators.

*see &lit clues below

Common types of word/letter play:

- **Anagram:** An anagram is a rearrangement of the letters of a word or words. This can be indicated by a wide range of words that imply motion, weirdness, craziness, or damage. Note that solving cryptic clues should be fun, and solvers are encouraged to use whatever tools they please, including online anagram solvers.

  Example 2 Flawlessly demonstrated n over p <u>is irrational</u> (6)

- **Letter selection:** Letter selection takes certain letters from the fodder. It is indicated with words like *firsts*, *lasts*, *middle*, *outers*, etc. depending on the fodder being selected.

  Example 3 Garden creature <u>starts to</u> go nuts — or mad even (5)

- **Letter placement:** Letter placement takes some part of the fodder and indicates where to place it within the answer. It is often used in conjunction with letter selection. These are indicated with words like *after*, *before*, *containing*, *contained within*, or *reversed* depending on where the fodder is to be placed.

  Example 4 Before or <u>after</u> pi <u>over</u> r? (5)

- **Deletion:** A deletion removes some fodder from some other fodder and, like the above, it is often used in conjunction with letter selection. They can be indicated with words like *loss*, *free*, *without*, etc.

  Example 5 A horizontal arrangement grows <u>without bounds</u> (3)

- **Hidden word:** Hidden word techniques have the letters of the answer in sequential order, either contained in a single word or spanning across multiple words. These can be indicated with words or phrases like *holds*, *hides*, *between*, *secretly*, etc.

  Example 6 The Extrema Theorem <u>reveals</u> what Euler studied (4)

- **Substitution:** A substitution is a replacement of some fodder with a commonly-used alternative. Substitutions differ from the other techniques in that they rarely have indicators and, as such, many people find them to be the most challenging technique to get the hang of. Substitutions include abbreviations like *female* $\mapsto$ f, *yes* $\mapsto$ y, or *Organization* $\mapsto$ Org; roman numerals like 55 $\mapsto$ LV; chemical symbols like *sodium* $\mapsto$ Na; shapes like *donut* $\mapsto$ O, *cross* $\mapsto$ x or t; cardinal directions like *north* $\mapsto$ n; phonetic alphabet like *Whiskey* $\mapsto$ W; and common synonyms like *feline* $\mapsto$ cat.

  Example 7 Well-kept grass is a countrywide rule, no? (4)

- **Homophone:** A homophone is a special type of substitution typically paired with an indicator such as *say*, *sounds*, *heard*, etc. where we find a substitution that sounds similar to our fodder.

  Example 8 <u>Reportedly</u>, $f(x)$ gives results (7)

- **Rebus:** Rebus clues are also often referred to as “say what you see” clues. These clues will often have something odd about them — maybe a misspelled word, a backwards word or phrase, use of alliteration, unusual capitalization, etc. The answer to these clues will not only fit the definition part of the clue, but will also explain the oddity.

  Example 9 Tuungsten (6-1)

- Note that some clues may also involve odd techniques unique to that clue. In this puzzle, these clues will be indicated with an asterisk.

- Other types of clues that differ from the standard structure are double-definition clues and &lit clues. In **double-definition** clues, rather than having definition and wordplay, we have two definitions back to back, and in **&lit clues**, the whole clue is both definition and wordplay!

  Example 10 Film studio complete collection? (3)

  Many, if not most, clues will use multiple techniques in the wordplay.

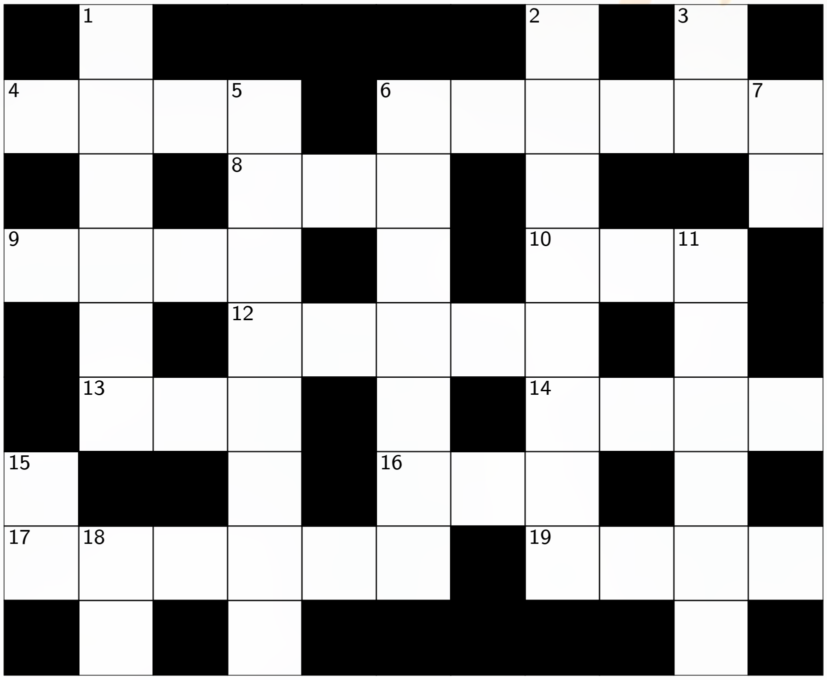
  Example 11 Georgia churned soil as a field worker? (6)

Answers:

Example 1: ROW	Example 2: PROVEN
Example 3: GNOME	Example 4: PRIOR
Example 5: ROW	Example 6: MATH
Example 7: LAWN	Example 8: EFFECTS
Example 9: DOUBLE-U	Example 10: SET

Example 11: GA + LOIS = GALOIS

Crossword Puzzle
By Nathaniel Alloway and Evan Socher



Across

4 Norwegiln mlthemlticiln (4)

6* Let X be uniformal. Then

$$X \begin{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} & \mathbf{0}_{1\times 5} \\ \mathbf{0}_{5\times 4} & I_5 \end{bmatrix}^T$$

is Gaussian. (6)

8 Long lost silver ring (3)

9* Write: $\frac{d}{dx}e^{xt}$? (4)

10 . . . plus—Dad's crazy! (3)

12 Calculator company with dark skinned godfather? (5)

13 Say, try root of 3? (3)

14 Like, "Set. . . Go!" perhaps (4)

16 Euclidean embedding lemma starts kind of fishy (3)

17 Look, a paper is complicated (6)

19 Face-to-face meeting held between renowned geometers? (4)

Down

1 Protest class member categorically! (6)

2 A three-sided contour integral is absurd (8)

3 Mother's force. . . by law of nature? (2)

5 All new iCast-LTE equipped with Meet and Join! (8)

6 German algebraist's exciting theorem replacing initial morphism with terminal projection (7)

7 Magical nymph reveals how to bring down powers naturally? (2)

11 Destroy arcane decagram without circular border (6)

15 Father Pascal (2)

18 First two odd primes greater than three less than four? (2)

You can play this puzzle online at https://tinyurl.com/forallpuzzle4-1. Solutions are available on our website.

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