



FORALL

SAINT LOUIS UNIVERSITY.

Beautiful Place.

Saint Louis is our home, and we endeavor to make it a better place by creating inclusive communities that can share in the joy of learning and the city's beauty.

ABOUT US

Biannual inclusive mathematics magazine featuring articles and expositions on a wide range of topics related to mathematics and more.



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Welcome to *For∇ll*

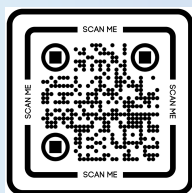
Welcome to another exciting edition of our math magazine For∇LL!

As always, we are thrilled to have you on board to explore a wide range of mathematical articles and expand our mathematical thinking together. In this issue, you'll find some applications of mathematics in computer science, machine learning, and computational graphics, along with insights into special subsets of permutations that have surprising combinatorial properties. Last but not least, enjoy our new Math in the Wild challenge and a new crossword puzzle created for our magazine!

Our main goal is to make math accessible to all, regardless of background. Whether you are an experienced mathematician or just starting your journey, we encourage you to explore, question, and engage with the content within these pages. This being said, we extend a warm invitation for submissions from all — undergraduate students, graduate students, and experienced researchers alike. Your contributions add variety to our journal, and we particularly encourage students to share their unique insights. Whether it is an undergraduate project or a groundbreaking research endeavor, we value the richness each perspective brings to our mathematical community.

Thank you for being a part of our journey, and we hope you find inspiration and enjoyment in the pages that follow.

Follow us and submit your content on our website!



forallmathmagazine.com

This is a journal of the people in our community, and it is strengthened by any art, puzzles, or articles the community can provide.

For∇ll Editorial Team

- Editor in Chief - Özlem Uğurlu, Ph.D., Assistant Professor at Saint Louis University
- Student Editor - Kieran Favazza, Undergraduate student at Saint Louis University
- Student Editor - Nathaniel Alloway, Undergraduate student at Saint Louis University

An advertisement for the American Mathematical Society (AMS) featuring a word cloud of math-related careers. The text "Shape your future with math—" is at the top in blue, followed by "power your career in technology, business, education, and beyond!" in blue. The word cloud includes careers such as AI engineer, urban designer, estimator, cryptanalyst, epidemiologist, technical writer, teacher, appraiser, actuary, claims adjuster, benefits administrator, population ecologist, biostatistician, quantitative analyst, stockbroker, smart city designer, virtual reality developer, game theorist, cybersecurity analyst, computer programmer, underwriter, banker, forensic analyst, climate analyst, production manager, commodities trader, market research analyst, statistician, animator, climate modeler, public utilities analyst, air traffic controller, foreign exchange trader, digital asset analyst, research scientist, financial aid director, professor, and data scientist. The AMS logo and the website www.ams.org/profession are at the bottom.

Faculty Interview Series

Interviewing: Fernando Piñero González

Interviewed by: Nathaniel Alloway and Kieran Favazza



Introduction

Mathematics constantly challenges us to be both broad and specific – to study things intensely, but study them in a way that can be communicated effectively. In this edition of our Faculty Interview Series, we welcome the wisdom of Dr. Fernando Piñero González, Associate Professor of Mathematics at the University of Puerto Rico in Ponce, and he shares with us his experiences and wise advice for understanding mathematics deeply while being able to educate and work with mathematicians at all levels.

Interview

Dr. Piñero, how did you become interested in mathematics?

Like many people, I've liked math since I was young, but when I entered my university studies, I wanted to go into medicine – I started out as a chemistry major. Organic chemistry quickly crushed that desire, but while that was happening, I was doing very well in calculus,

and someone told me there was a math course without any numbers. That was abstract algebra, and I fell in love with it. I was a little hesitant to pursue this since I didn't know what I could do with a math degree, but I knew that I could always get my degree and go to work with my father who worked in the HVAC business. But luckily, I was able to find math-related employment.

Algebra can often turn people off of math - how did algebra make you more excited about math?

The exact thing that can turn people off – the fact that it can be so weird. For example, we are used to usual operations on numbers – like how 2×3 is the same as 3×2 , but you start encountering things like the quaternions and weird places where you can do division and multiplication, but it's not commutative – these kind of weird things are the things I really love – I'm grateful I get to study such weird things. I was fortunate to experience great mentorship in algebra in my graduate studies, facilitated by the Louis Stokes Alliance for Minority Participation (LSAMP).

Would you say that finding peculiar things is a pattern in your life? How do strange things appear in your research?

I do a lot of work in finite fields – mostly in coding theory – an area about which I run a Research Experience for Undergraduates (REU). After having fallen in love with algebra, I didn't know it could have such real-world applications; in my research, I use the nice properties of fields and all the tools of linear algebra and polynomials to enhance and study error-correcting codes. For example, a very elementary kind of encryption transmits points in space to encode polynomials – I enhance this with the study of curves and how polynomials relate to them, using the high-powered tools of algebraic geometry to go back and forth between geometric information, algebraic data, and codes. One of my key contributions in this area has been to communicate this machinery with very

concrete language – simple enough so that one does not need extensive training to do meaningful work. This is one of the reasons our REU has been so successful, and I find great pleasure in distilling complex information into manageable and accessible methods.

What are some ways to communicate advanced topics in a simple manner that are particularly effective?

Something I've come to realize since becoming a professor is that we are not just teachers, but facilitators. It's not so much that you are giving information to someone, but you are being there so that you can be part of their learning experience. There are many experiential aspects of learning that hinge on the effort of the student, so I do my best to adapt and find activities that are close to the level of my students' understanding. It is certainly a process of trial and error, as all teaching is, but it's important to remember that learning is primarily a social activity. So it's very key that I relate to students as best I can, both as a person and a mathematician. I do my best to meet students at their level – finding and presenting aspects of projects and concepts to suit students' interests and backgrounds.

How did the LSAMP program help you in pursuing graduate studies?

The LSAMP program is established by Congress, and schools in LSAMP can apply to the NSF for a Bridge to the Doctorate (BD) grant. Students in LSAMP who are starting out their doctoral studies can receive funding through these grants. When I was graduating from my undergrad, my home institution had received a BD scholarship. However, the program is in general science, and there they had very little math people. So, the coordinator said, "Fernando, what are you doing next year?" I thought I would maybe take a year off or go into industry, but he told me, "No, no, no, you're going into the LSAMP Bridge to the Doctorate scholarship." When I said the deadline passed, he said he'd talk to the dean, and it all worked out.

Such strong mentorship is so wonderful and key in mathematics – as part of my REU, I bring fellow professors and graduate students to talk about their personal experiences in grad school. A common theme I can see is that they all had support. They all had someone cheering them on, saying, "You can do it, you have to do it." So my advice for

students is to get support, and for professors is to support as many students as they can.

Do you have students that you are close with?

I do and I don't. At my home institution, I am in a service program, so most of the students I see right now are students who would later transfer to a different engineering school. We see each other and say hi from time to time, but there aren't any undergraduate students I can mentor directly.

However, I can impact students in other ways. Because we live in the Information Age, there's a big push for computation and statistics and such in biology. So I'm making some headway with individuals in biology. When teaching students math, I also bring up examples of biology, and their eyes light up. I'm able to mention things like analyzing the frequency of the croak of the *coquí* (a tree frog emblematic of Puerto Rico), and I'm able to engage students in a way they like and connect with them.

So you've been learning a lot about biology, then?

Yes. Actually, since the LSAMP program was skewed so heavily toward biology, I learned many things about conservation and whatnot during graduate study. But you know, it's our environment, and we should take care of it. The role of academia should be similar: learning in service to humanity.

Are there ways you implement this ideal in your teaching/work?

I mean, if you teach well, you are already doing this! Teaching is a way you can help the next generation of humanity incorporate a fine corpus of knowledge. I'm also working on establishing more collaborations with the biology department. There, they have loads of data, and it's tough to analyze. We have machine learning techniques for classifying, say, birds and amphibians, for developing a biodiversity index. So this is another way I try to help.

How else could you tie outside sciences into math?

Try the following: Take a small sample of your undergraduate colleagues in math, and ask them what they know about the *t*-test in itself. Then see if they know any ways to apply it, e.g. to measure drug efficacy or noise pollution. If you find something significant, let me know!

For me, there are so many biology and biomedical students at UPR Ponce that, really, the biology examples light my students up.

What is your life outside of math? Do you have things you enjoy?

Always! If you don't, you go crazy. At this stage, my life mostly revolves around my wife and children. While we're here, let me pass on some fatherly advice from my elders: time is short and it flies by quickly. Before you realize it, your cute children will grow up and become surly, so enjoy your time with them while you can! So right now I just take my children swimming or to the movies, and when they're older we'll do other activities.

I'm also working on my work-life balance – hitting the gym and doing yoga. But other than that, just life. I am lucky to have grown from a passionate youth to the kind of life I have today.

What are some ways we can navigate the work-life balance?

Here's another bit of advice I learned: You cannot pour water from an empty cup. It's not that you have to be selfish, but you do have to take care of yourself first. Don't push yourself above and beyond what is healthy. Maybe an all-nighter or two, but they should be few and far between. Also take time to celebrate the victories, especially the small ones. If you don't, everything sort of becomes dull, and you want to be happy and engaged with everything.

Overworking yourself can be like spinning your wheels: You expend all this effort just to go nowhere. So you also need to take time out to take care of yourself – enough sleep, enough rest, and so on. Overall, the most important part is to be happy.

You make an excellent point about prioritizing your well-being; and celebrating the small stuff. Mathematicians, especially, tend to forget this – do you think it's the math or the mathematicians? Do people who put pressure on themselves become mathematicians, or is math the kind of subject that causes people to put pressure on themselves?

Something else to think about is the environment. Some places are more laid back, other places less so. Different people do well in different places. One of the reasons why I studied back home was

because I knew that my home institution and I got along.

Something else to consider: I took great advantage of the on-campus counseling. I have a curious mind, so I always have to ask and ask. So whenever I was struggling with something, I would go to the counselors and ask my questions to them. If you have counseling services available... avail yourself of them!

More advice (for math research, for finding a spouse, for anything): If it were easy, it'd be done already! (There's this research problem that's still around from 1978... gee, I wonder why!)

Do you have any advice for helping our colleagues and friends take care of themselves better? How can we be more supportive?

Having a space where people can come in and discuss things freely – formally or informally – is key. At my Ph.D. institution, we had a strong coffee-time culture. We would sit and eat pastries and talk about things. If you don't have something like this at your own institution – be the change you wish to see in the world! Pick the best time possible, and try to gather people – even just 3 people can be a crowd. Word will spread, and you'll have more participants. Also consider reaching out to other departments (e.g. physics, biology, or social sciences for something harder). They'd have additional resources and people to form support networks.

Another thing: Something problematic about academia is that there's a conflagration between your job and your identity. Your self-worth shouldn't be tied to your job, but this is more difficult for a teacher! To the extent that you can, try to keep things separate. Yet, also know that you can't teach without passion! Students can tell when you're uninterested.

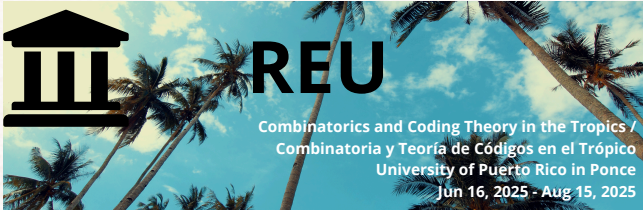
Conclusion

Dr. Piñero effectively uses his mathematical and life experiences to excel as a mathematician and educator. He constantly evolves his understanding and methods of communication, and he reminds us of the importance of a well-balanced life. He balances his interests with those of his students, and he balances his personal experiences with his professional ones. We are grateful for Dr. Piñero for

his wonderful advice on how to navigate the many difficult and exciting tasks in mathematics!

Advert

Dr. Piñero is hosting an REU this summer at the University of Puerto Rico in Ponce. For more details, please see the posting at <https://etap.nsf.gov/award/1058/opportunity/10669>.



REU SITE: COMBINATORICS AND CODING THEORY IN THE TROPICS
REU: COMBINATORIA Y TEORÍA DE CÓDIGO EN EL TRÓPICO



Fernando Piñero González
 University of Puerto Rico in Ponce



Pamela E. Harris
 University of Wisconsin at Milwaukee

Pamela and Fernando invite you to the REU: "Combinatorics and Coding Theory in the Tropics, sponsored by NSF and UPR-Ponce to be held from June to August 2025. Pamela and Fernando will lead several research projects in Combinatorics, Coding Theory, Graph Theory and others. We seek applicants from varied backgrounds (both academic and personal) to build a diverse research community. We offer a \$5,400 stipend, lodging and travel. Students from underrepresented groups and females are strongly encouraged to apply.

Pamela y Fernando le invitan al REU: "Combinatoria y Teoría de Códigos en el Trópico" un REU apoyado por NSF y UPR-Ponce. Se llevará a cabo en Puerto Rico desde junio hasta agosto de 2025. Pamela y Fernando liderarán varios proyectos de investigación en combinatoria, teoría de códigos, teoría de grafos y otros. Buscamos solicitantes de trasfondos variados (académicos o personales) para construir una comunidad de investigación diversa. Ofrecemos un estipendio de \$5,400, viaje y alojamiento. Exhortamos fuertemente a aspirantes de grupos subrepresentados y féminas a aplicar.

For more information visit // Para mas información visite:
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 To apply please visit\\Para solicitar visite:
<https://etap.nsf.gov/award/1058/opportunity/10669>
 email: fernando.pinero1@upr.edu
 Deadline: April 12, 2025 \\ Fecha Límite: 12 de abril de 2025




Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called "EvenQuads" to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.



Emmy Noether
(1882-1935)

Noether was born in Erlangen, Germany. She taught for a quarter century there and in Göttingen, often without pay or credit. Noether was the first female plenary speaker at the International Congress of Mathematicians (1932). Like many Jews, she left Germany following the Nazis' rise to power. She moved to Bryn Mawr College (1933), but died within three years. Her considerable legacy includes Noetherian rings in abstract algebra and the Noether Theorem in theoretical physics. Noether supervised 16 doctoral students, two of whom were women. The Emmy Noether Lectures were established by the AWM to honor women who have made significant contributions to mathematics.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.



Sellenne Bañuelos
(b. 1985)

Sellenne Bañuelos obtained her PhD from USC (2013). She is currently an associate professor at California State University Channel Islands (CSUCI) and associate director at the Institute for Pure and Applied Mathematics (IPAM). Bañuelos was on the leadership team of Project PROMESAS, a five-year, \$6 million Hispanic-Serving Institution STEM grant. Bañuelos was an organizer or participant at the IPAM Latinx in the Mathematical Sciences Conferences in 2015, 2018, and 2022. She serves as an advisor to the CSUCI SACNAS chapter, and does mentoring through the national Math Alliance and the CSU Alliance PUMP program. Bañuelos received a 2020 MAA Henry L. Alder Award for distinguished teaching.

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An Introduction to Path Tracing

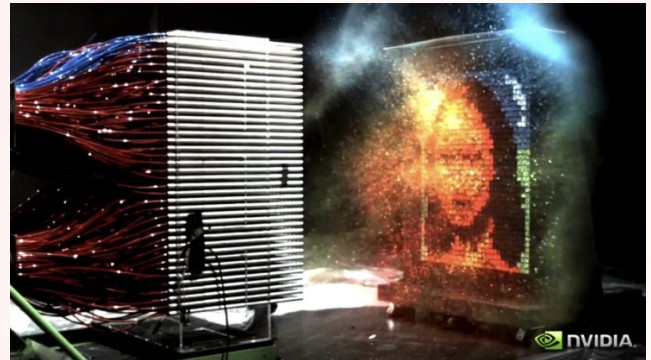
By Michael Teschner

Undergraduate Student, Department of Computer Science, Saint Louis University

The History of the GPU

Computer graphics is a subject in computer science that, while being an antique field of research, remains largely shrouded in mystery and complexity. The concepts behind computer graphics are relatively simple and consist of the programming techniques, data structures, and mathematical models required to create a 3D representation of data and present it on a 2D screen. Another way to phrase this definition is that computer graphics is the study of converting point data, or vertices, into a 3D mesh and drawing it on a computer monitor in a process known as rasterization.

Though the techniques used to draw vertex data to a computer screen have not changed drastically since their conception, [2], the hardware used in rasterization has changed dramatically with the introduction of the GPU in 1999 by NVIDIA, [1]. A GPU, short for Graphics Processing Unit, is a piece of hardware similar to the CPU (Central Processing Unit) of a computer that uses transistors and binary logic to perform computations. Where the GPU differs from the CPU is in its ability to perform simple calculations many times in parallel, while the CPU is better suited for performing a single complex calculation. This new piece of hardware was instrumental in the development of computer graphics, as until then all calculations were done on the CPU. In an attempt to explain why this was inefficient, I will defer to an example provided by Mythbusters in 2009, [3]. The Mythbusters team explains how the CPU has to iterate through each pixel on the screen and calculate its color sequentially, while the GPU can process each pixel on the screen in parallel. They demonstrated this concept by an elaborate mechanism in which a grid of paintball guns shoot at a blank canvas in an attempt to paint the Mona Lisa. The CPU demonstration shot each paintball one at a time, slowly painting the Mona Lisa, while the GPU fired



Adam and Jamie Paint the Mona Lisa in 80 Milliseconds!
Credit: [YouTube](#)

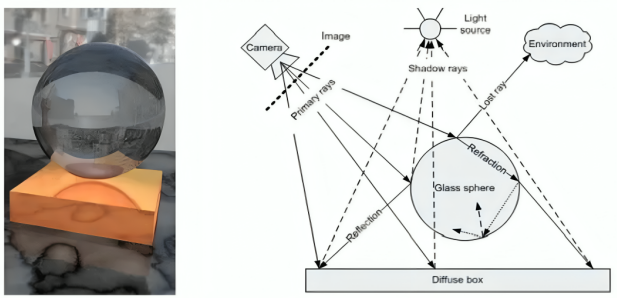
the entire grid of paintball guns at the same time, painting the Mona Lisa instantly.

The introduction of the GPU progressed research in the fields of rendering and image processing, while also contributing to one of the most technically challenging topics in computer graphics: global illumination.

Realtime Global Illumination and Path Tracing

Global Illumination is the task of realistically simulating lighting and the behavior of light in a 3D scene. There have been many different approaches to this task, but among the most successful approaches is a method known as path tracing. To understand how path tracing works, we must first understand how light works in the real world. Light is emitted from a source and bounces off objects, which absorb certain wavelengths of this light until it reaches our eyes. We perceive different colors of light depending on which objects the photons bounce off during their transit to our eyes. Simulating this on a computer would be terribly inefficient as the vast majority of light emitted from a source never reaches our eyes. To visualize this, consider the sun. The sun emits an unimaginable amount of

photons in all directions and only a minuscule fraction of these photons are lucky enough to even reach our planet. Of these lucky few photons, even less will reach the eyes of an individual person. To simulate realistic lighting on a computer, path tracing uses the concept of a ray bouncing around a scene, like a photon, only with one major difference: instead of the origin of the ray being the light source, the origin is the “eyes” of the computer screen.

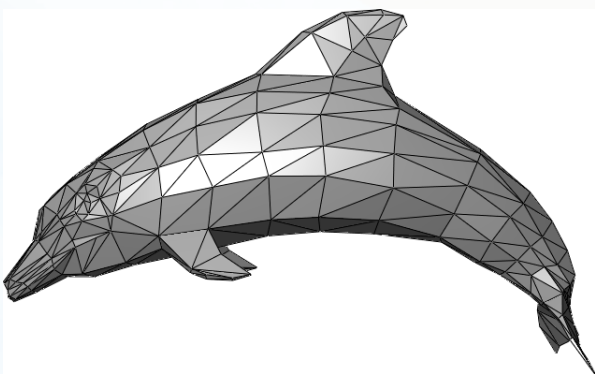


Ray-traced image (left) and visualization of contributing light rays (right). Credit: Louis Young [4]

Each pixel on the screen emits a ray into the scene and keeps track of which surfaces the ray bounces off, coloring the pixel based on this data.

Path Tracing in More Depth

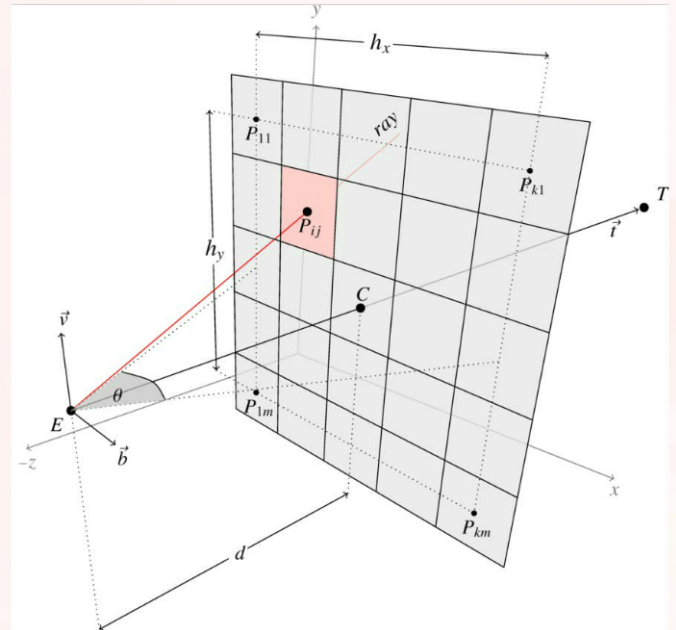
The individual steps involved in path tracing, though intensive in linear algebra, are not particularly difficult to understand. The only prerequisite knowledge required is the understanding that all 3D objects in computer graphics are represented as triangles, due to their simplicity to render.



Example of a triangle mesh representing a dolphin. Credit: Wikimedia Commons

The first step in path tracing is to emit a ray from each pixel on the screen, from the origin of the camera. The camera is represented by a 3D vector for its position and a 3D vector for its direction. The ray is sent in the forward direction of the camera,

with a slight offset depending on the location of the ray’s origin pixel.



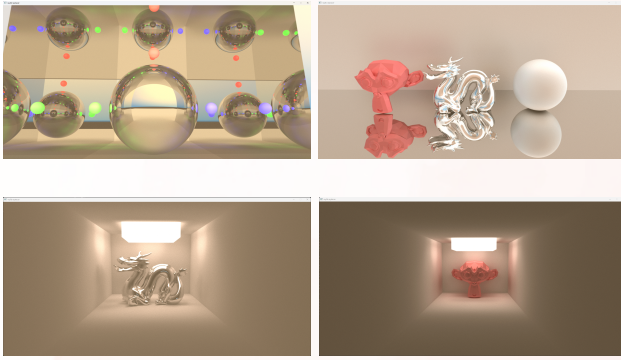
Viewport schema with pixels, eye E and target T , viewport center C . Credit: Kamil Kielczewski, Wikimedia Commons

The second step in path tracing is to iterate through each triangle in the scene and check if any rays collide with it. If the ray does not collide with anything in the scene, it is colored based on the “skybox,” which is a texture used for the background of a scene. If the ray does collide with a triangle, it stores the color of the triangle it hits and bounces using the normal vector to the triangle. The ray is then shot again, only this time from its collision point with the triangle, and in the bounce direction. Lighting is calculated by giving each triangle a “light” value, or emission value as it is called in computer graphics, and calculating the intensity of the pixel’s color based on this value.

This process of bouncing and calculating lighting data repeats until the ray either runs out of bounces, which we specify in code so that it doesn’t bounce an infinite number of times, or until it does not collide with any triangles. The result is a nearly photorealistic image of a 3D model. To show this process in action I will provide screenshots of a C++ path tracer I wrote over this winter break using the Raylib library and the GLSL shader language.

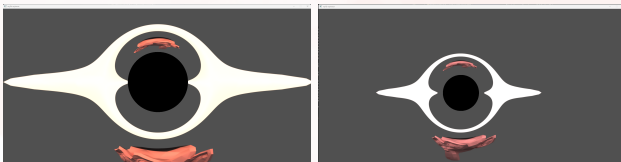
Conclusion and Further Research

The concept of path tracing opens the door to many fascinating fields of research, such as black hole



Images of my software path tracer creating realistic lighting in 3D scenes

rendering, which accounts for relativity when calculating ray directions. To provide an example of this, I modified my existing path tracer code to curve rays around a singularity, which resulted in the following images:



An early attempt at simulating singularities using my existing path tracer code

Another very active field of research is that of real-time path tracing. This field is devoted to reducing the computation needed to perform path tracing since even though the results of path tracing are extremely realistic, they have the drawback of taking up to several days to compute. These are just a few examples of the many fields of research devoted to lighting simulation.

Though the topic of computer graphics is complex and requires large amounts of research to put into practice, it is a topic that I find to be one of the most fascinating aspects of computer science. Its contributions to video games, physics simulation, CGI, and cinematography have greatly impacted the world and I believe it will continue to do so for years to come. I hope that by writing this article I have been able to display why I find this topic to be so interesting and also have helped remove the shroud of complexity surrounding it. If you are interested in learning more about my project or collaborating, here is a link to my GitHub account as well as the project source code: <https://github.com/BlazeTechDev/RaylibRaytracer>.

References

- [1] Jake Frankenfield. “Graphics Processing Unit (GPU)”. In: *Investopedia* (Sept. 2021). Accessed: 2025-02-12. URL: <https://www.investopedia.com/terms/g/graphics-processing-unit-gpu.asp>.
- [2] LearnWebGL. 1.3 - *Computer Graphics - a Brief History*. Accessed: 2025-02-12. URL: https://learnwebgl.brown37.net/the_big_picture/webgl_history.html.
- [3] Mythbusters. *Mythbusters Demo GPU versus CPU*. Accessed: 2025-02-26. URL: <https://www.youtube.com/watch?v=-P28LKWTzrI>.
- [4] Louis Young. “An Overview of the Path Tracing Pipeline”. In: *Medium* (Apr. 2018). Accessed: 2025-02-12. URL: <https://medium.com/@youngalou/an-overview-of-the-path-tracing-pipeline-eefec92aad35>.

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Julia Maria Aguirre
(b. 1966)

Julia Maria Aguirre earned her BA in Psychology from the University of California, Berkeley in 1989, her MA in Education from the University of Chicago in 1993, and her PhD in Education from the University of California, Berkeley in 2002. She is Full Professor of Education at the University of Washington, Tacoma. Aguirre's research is in helping educators provide accessible, relevant mathematics education, bridging gaps across backgrounds, cultures, languages, beliefs, and practice. She is co-Principal Investigator on the NSF grant “Teachers Empowered to Advance CHange in Mathematics” (TEACH Math), the co-author of the highly cited 2013 book *The Impact of Identity in K–8 mathematics: Rethinking equity-based practices*, and the author or co-author of eight book chapters.

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Visualizing Machine Learning by Classifying Number Images with a Fluid Simulation

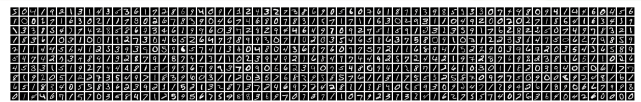
By Jack Crane

Undergraduate Student, Department of Computer Science, Saint Louis University

I saw an incredible video on YouTube about someone who built a computer using just the flow of water [3], and the creator demonstrated how he set up gates and was able to string them together to form a 4-bit adder. It was an incredible demonstration of the power of simply pouring water and its natural tendency to flow in all directions but up. This got me thinking about how far we could push water to solve problems. If he was able to create XOR and AND gates, then naturally we could create the other common gates and build a computer that could solve any problem (albeit very slowly). This brought me back to my introduction to Electrical Engineering and Fluid Dynamics courses and it really got me thinking about how almost everything in Fluid Dynamics and Thermodynamics are just rephrasings of electrical engineering problems. Insulators are just resistors, flow is just a current, and so on. I began seriously thinking about how I could create a more significant computer using water, and using what I know about fluid dynamics and electrical/computer engineering, I began to think about how I could build a larger system. The more I thought about it, however, the reality of being a college student without access to enough room, time, resources, or energy to build a project like this began to set in. While up late studying for my Calculus III and Fluid Dynamics final exams, it hit me that fluid flows around different objects differently, and if I could simulate the fluid flow around an object, I could find a way to store information about the simulation and use some vector nearness to find the nearest neighbor, and hopefully recognize the object. This article is the story of how I did that.

Like any good project, the first step was to define the problem. I suppose that this would be some kind of AI classification problem. Although I have never worked with AI and computer vision before, I do know that building a number classifier is a

very common first project, so I decided to start with that. There is a dataset published by the National Institute of Standards that provides thousands of samples of handwritten digits, colloquially known as the MNIST dataset. I downloaded the MNIST dataset [2] and extracted the images, and then it hit me how insane the project I had taken on was. So I started researching to find if someone had any ideas that I could bounce off of and found another YouTube video [4] of someone doing almost exactly what I was trying to do.

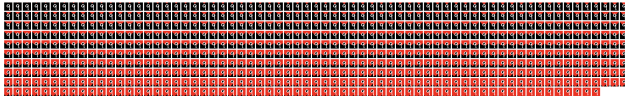


A sample of the MNIST dataset

After processing the images, I began processing the data. I had to find a way to represent the image data in a way that I could convert into an n -dimensional vector. I began thinking about how, given (for example) the number 3, pouring water over the top would allow all the water to flow downwards and none would be captured. However, if I rotated it 90 degrees clockwise, the water (pouring from the new top) would be captured in the curved bits of the 3. If I recorded how much water was captured in each orientation, I could use that to create a vector to group common features together.

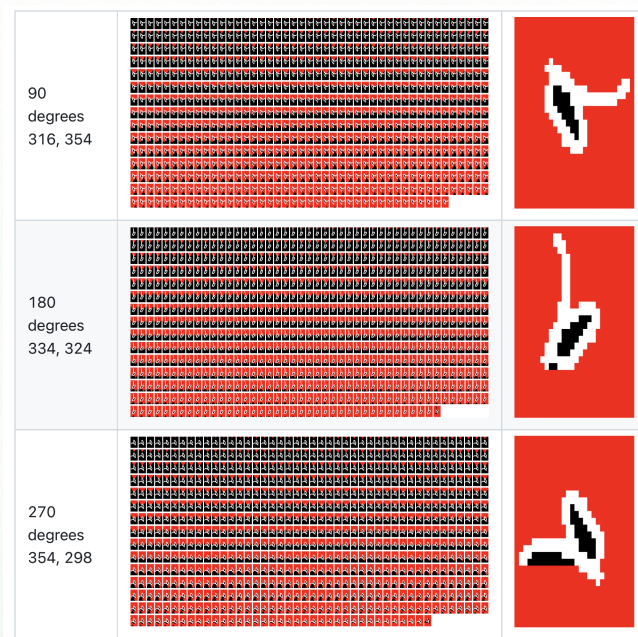
The more I considered it, I began to realize the issues I would have distinguishing between numbers like 6 and 9, seeing that they have similar features in different orientations and areas. After watching [4], I realized that if I could break the retained and blocked water into 8 regions — the left and right halves of the original, 90-degree clockwise, 180-degree, and 270-degree rotations — I would be able to embed not only the presence of features but

also the general position of said features.



Frames of a fluid simulation pouring over the number 9

After a great deal of messing around, I was able to convert numbers in different orientations to two values, representing the number of filled pixels in the left and right halves of the image. Using the core idea that water is able to flow down, left, and right, and is not able to flow up or through barriers, I generated a vector that represented the presence of the number in each orientation.



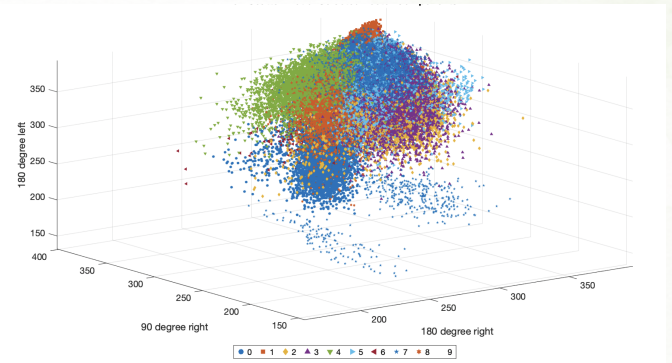
Frames of a fluid simulation pouring over the number 9 in the 3 rotated states

If you observe the frames or watch a video of the simulation [1], you will notice that the simulated fluid's flow is remarkably similar to the behavior of a breadth-first search algorithm. The fluid does not flow in a way that is consistent with either a depth-first or breadth-first search, but in an open control volume. It tends to expand in all directions with a consistent velocity, leading to a similar flow pattern to a breadth-first search.

Given the flow patterns above, we can construct a 9-dimensional vector that represents the total number of fluid pixels, where the search algorithm is allowed to propagate in all directions rather than just three, followed by the number of filled pixels on both halves of each of the four orientations. This

gives us an understanding of both the features of the digit, but also the size of any enclosed spaces.

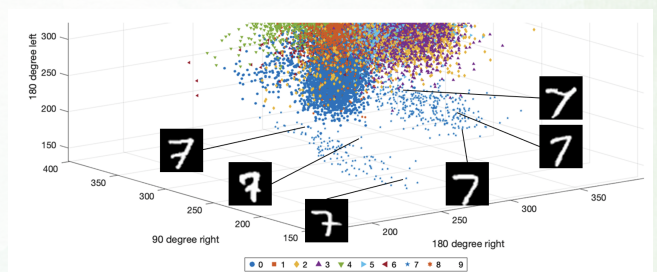
I then ran the “training” script which calculates this vector for each image in the dataset, and got over 60,000 vectors. While it is a 9-dimensional vector, we can reduce it to a 3D graph by picking 3 components to plot. For the sake of this article, I plotted the left halves of 180-degree and 90-degree turned images, and the right half of the 180-degree image.



A 3D scatter plot of the MNIST dataset having been processed with the fluid simulation.

Looking at this scatter plot, we can see how different characters are grouped together. For example, the cluster of lime green downward-facing triangles is a huge collection of 4 characters.

There are some disparities in the distribution of some of these sample. For example, the 1 character is in a very tight cluster with other 1 characters since there are not that many ways to draw a 1. On the other hand, 7 has two distinct clusters along the bottom, which is an indicator of the differences between how the character can be drawn, with one band being a normal 7, and the other being a 7 with a diagonal line through it.



A 3D scatter plot of the MNIST dataset having been processed with the fluid simulation, connecting some of the 7 data points to the images that created them.

From our 9-dimensional point cloud, the last step is to identify the closest neighbor to a new, unseen

number. We can do this using the Euclidean distance formula of an n -dimensional vector:

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}.$$

Now, given a new, unseen image of a number, in the correct format, we can extract features from it, evaluate it, and connect it to other similar characters in the dataset.

There are far more efficient and accurate ways to do fluid simulations using deep neural networks, but using such a tangible approach to the problem was extremely interesting to me and I wanted to share it.

References

- [1] DigitalOcean Spaces. Accessed 2024-02-19. 2024. URL: <http://jack-general.nyc3.digitaloceanspaces.com/floodfill-limited-270.mp4>.
- [2] Kaggle. Accessed 2024-02-19. 2024. URL: <http://www.kaggle.com/datasets/hojjatk/mnist-dataset>.
- [3] YouTube. Accessed 2024-02-19. 2024. URL: <https://youtu.be/IxXaizglscw>.
- [4] YouTube. Accessed 2024-02-19. 2024. URL: http://youtu.be/CC4G_xKK2g8.

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Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called “EvenQuads” to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.



Grace Alele-Williams
(b. 1932)

Alele-Williams is Nigerian, and spent many years as professor of mathematics education in the University of Lagos before becoming the first woman Vice Chancellor of any African university (at the University of Benin). Her PhD from the University of Chicago (1963) made her the first Nigerian woman to earn a doctorate in any subject. She is recognized as a visionary and effective leader in education in Nigeria, and her numerous awards include the Order of the Niger (1987), the Merit Award of Bendel State in Nigeria, and being elected a Fellow of the Mathematical Association of Nigeria and the Nigerian Academy of Education.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.



Christine Mann Darden
(b. 1942)

Christine Mann Darden earned an MS in Applied Mathematics at Virginia State University after graduating from Hampton College with a BS in Mathematics. She obtained a PhD in Mechanical Engineering from George Washington University in 1983. Darden was an aerospace engineer, researcher, data analyst, and leader at NASA's Langley Research Center. She joined NASA in 1967 as a “human computer” and became one of the first female engineers there, developing the sonic boom research program at NASA. She became the first Black woman promoted into the top rank of the federal civil service at Langley, and was awarded the Congressional Gold Medal in 2019. An MAA lecture series is named in her honor.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.



Descent Polynomials

By Martha Precup

Associate Professor, Department of Mathematics, Washington University in St. Louis, Email: martha.precup@wustl.edu

Permutation groups are simultaneously approachable, amenable to computation, and mathematically rich. Let n be a positive integer and $[n] := \{1, 2, \dots, n\}$. The *symmetric group* is the set of all bijections from $[n]$ to $[n]$ with group operation equal to function composition, denoted by S_n . The elements of the symmetric group are called *permutations*.

How many elements are in the set S_n ? If $\sigma \in S_n$ then we have n choices for the value of $\sigma(1)$, then $n - 1$ choices for the value of $\sigma(2)$, and so on. It follows that there are a total of $n!$ elements.

Consider the permutation $w \in S_5$ such that $w(1) = 2$, $w(2) = 3$, $w(3) = 1$, $w(4) = 5$ and $w(5) = 4$. We can write

$$\begin{aligned} w &= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ w(1) & w(2) & w(3) & w(4) & w(5) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{bmatrix} \end{aligned}$$

Notice that the second line 23154 in the array above completely determines the permutation w . For example, the fact that 3 appears second in this list tells us that $w(2) = 3$. We call 23154 the *one-line notation* of w . There are many other ways to denote permutations, but we stick with one-line notation in this note.

Any two permutations can be composed to give another permutation. We call S_n a group because composition defines an associative binary operation on the set of all permutations, every permutation has an inverse, and the permutation $e = 12 \cdots n$ is the *identity*, that is, $ew = w = we$ for all $w \in S_n$.

Example 1 Let $w = 23154 \in S_5$. Then inverse of w is $w^{-1} = 31254$.

Although it is simple to define, we should expect the structure of the symmetric group and enumer-

ative properties of particular subsets to be very interesting! For example, Cayley's Theorem states that every finite group is isomorphic to a subgroup of the symmetric group S_n for some n . So, while computing with permutations is straightforward, we should not be surprised when we unearth new formulas or patterns. After all, the theory of finite groups is varied and rich!

Descents and Inversions

Let $w = w(1)w(2) \cdots w(n)$ be a permutation, written in one-line notation. An *ascent* of w is a position i such that $w(i) < w(i + 1)$ and a *descent* of w is a position i such that $w(i) > w(i + 1)$. The *descent set* of w is

$$\text{Des}(w) = \{i \in [n - 1] \mid w(i) > w(i + 1)\}.$$

Note that the descent set of $w \in S_n$ is a subset of $[n - 1]$. An *inversion* of w is an ordered pair (i, j) such that $1 \leq i < j \leq n$ and $w(i) > w(j)$. Let

$$\text{Inv}(w) := \{(i, j) \mid i < j, w(i) > w(j)\}$$

denote the set of inversions of w . Two important statistics on permutations are the number of descents and the length, or number of inversions.

Definition Let $w \in S_n$. The *length* of w is

$$\ell(w) := |\{(i, j) \mid i < j, w(i) > w(j)\}| = |\text{Inv}(w)|$$

and the *descent number* of w is

$$\text{des}(w) := |\{i \in [n - 1] \mid w(i) > w(i + 1)\}| = |\text{Des}(w)|.$$

Example 2 The permutation $w = 36281475$ in S_8 has descents in positions 2, 4, 7 so $\text{Des}(w) = \{2, 4, 7\}$. We have

$$\begin{aligned} \text{Inv}(w) &= \{(1, 3), (1, 5), (2, 3), (2, 5), (2, 6), (2, 8), \\ &\quad (3, 5), (4, 5), (4, 6), (4, 7), (4, 8), (7, 8)\}. \end{aligned}$$

In this case $\ell(w) = 12$ and $\text{des}(w) = 3$.

Example 3 Let $n = 3$. The following table lists all permutations in S_3 with the specified descent set.

$S \subseteq \{1, 2\}$	$w \in S_3, \text{Des}(w) = S$
$\emptyset$	123
$\{1\}$	213, 312
$\{2\}$	132, 231
$\{1, 2\}$	321

Descent Polynomials

Let I be a finite subset of positive integers and $m := \max(I \cup \{0\})$. For each $n > m$ consider the set

$$D(I; n) = \{w \in S_n \mid \text{Des}(w) = I\}$$

of all permutations in S_n with descent set I , and its cardinality

$$d(I; n) := \#D(I; n).$$

MacMahon proved that $d(I; n)$ is always a polynomial in n , [8]. We call $d(I; n)$ the *descent polynomial of I* . One straightforward example is the case of $I = \emptyset$. There is only one permutation with no descents, namely the identity. Thus, $d(\emptyset; n) = 1$ for all n .

Example 4 Consider $I = \{2\}$ and let $n > 2$. A permutation $w \in S_n$ has a unique descent in position 2 if and only if

$$w(1) < w(2), w(2) > w(3), \text{ and} \quad (1) \\ w(3) < \dots < w(n).$$

In particular, given any subset $\{k, \ell\}$ of $[n]$ with two elements, consider the permutation whose one-line notation starts with k, ℓ (where $k < \ell$) and then lists all remaining elements of $[n]$ in increasing order. For example, if $n = 6$ and $\{k, \ell\} = \{2, 4\}$ then $w = 241356$. Any permutation of this type will satisfy the conditions of (1) as long as ℓ is larger than the smallest element of $[n] \setminus \{k, \ell\}$. This will be the case unless $k = 1$ and $\ell = 2$. Thus,

$$d(\{2\}; n) = \binom{n}{2} - 1 = \frac{n(n-1)}{2} - 1 \\ = \frac{1}{2}n^2 - \frac{1}{2}n - 1.$$

which is indeed a polynomial.

A similar argument to the one appearing in the example yields,

Lemma 1. Let k be a positive integer. For all $n > k$,

$$d(\{k\}; n) = \binom{n}{k} - 1 = \frac{n(n-1) \cdots (n-k+1)}{k!} - 1.$$

In the lemma and example above, we obtain formulas using binomial coefficients. When we write polynomials, we are used to expressing them using the standard monomial basis of $\{1, x, x^2, x^3, \dots\}$. Since we are interested in enumeration we can instead use the *binomial basis*,

$$\left\{ \binom{x}{0}, \binom{x}{1}, \binom{x}{2}, \binom{x}{3}, \dots \right\} \text{ where}$$

$$\binom{x}{k} := \frac{1}{k!} x(x-1)(x-2) \cdots (x-k+1).$$

Example 5 The polynomial $\frac{1}{6}(x^3 - 3x^2 - 10x)$ can also be written as $\binom{x}{3} - 2\binom{x}{1}$.

Given a subset $I \subseteq [n-1]$, Diaz-Lopez, Harris, Insko, Omar, and Sagan in [5] gave an explicit formula for the descent polynomial $d(I; n)$. In order to understand their theorem, we need some terminology. Given two positive integers $k < \ell$, let $[k, \ell] = \{k, k+1, \dots, \ell\}$. Recall that m denotes the maximum value in the set $I \cup \{0\}$. Now, for each $k \geq 1$ we define

$$a_k(I) := \#\{w \in D(I; 2m) \mid \{w(1), \dots, w(m)\} \\ \cap [m+1, 2m] = [m+1, m+k]\}.$$

In other words, $a_k(I)$ counts the number of permutations in S_{2m} with a descent set I such that $\{m+1, \dots, m+k\}$ are precisely the elements greater than m that appear in the first m entries. The following theorem tells us how to compute the descent polynomial $d(I; n)$ for any subset I in terms of the integers $a_k(I)$ [5, Theorem 3.3].

Theorem 2 (Diaz-Lopez, Harris, Insko, Omar, Sagan). For any finite subset of positive integers I and $m = \max(I \cup \{0\})$ we have that

$$d(I; n) = a_1(I) \binom{n-m}{1} + \dots + a_m(I) \binom{n-m}{m}$$

and moreover, $a_k(I) > 0$ for all $1 \leq k \leq m$.

One beautiful consequence of this formula is that the coefficients of $d(I; n)$ are positive integers when expanded in the shifted binomial basis. In fact, for any subset I , the sequence $(a_k(I))$ is *log-concave*, which means that $a_{k-1}(I)a_{k+1}(I) \leq a_k(I)^2$; see [1]. Log-concave sequences appear naturally in combinatorics, algebra, and geometry. See [3] for more.

Let's compute an example of Theorem 2.

Example 6 Consider the set $I = \{1, 3\}$. In this case, $m = 3$ so the theorem tells us

$$d(I; n) = a_1(I) \binom{n-3}{1} + a_2(I) \binom{n-3}{2} + a_3(I) \binom{n-3}{3}$$

where

$$\begin{aligned} a_1(I) &= \#\{w \in D(I; 6) \mid \{w(1), w(2), w(3)\} \\ &\quad \cap \{4, 5, 6\} = \{4\}\} \\ a_2(I) &= \#\{w \in D(I; 6) \mid \{w(1), w(2), w(3)\} \\ &\quad \cap \{4, 5, 6\} = \{4, 5\}\} \\ a_3(I) &= \#\{w \in D(I; 6) \mid \{w(1), w(2), w(3)\} \\ &\quad \cap \{4, 5, 6\} = \{4, 5, 6\}\}. \end{aligned}$$

We can calculate these values easily using a computer. For example, if we ask for all permutations in S_6 with the descent set $\{1, 3\}$ and so that 4 is the largest element within the first three entries we obtain,

$$214356, 314256, 324156, 413256, 423156, \quad (2)$$

so $a_1(I) = 5$. Similarly, $a_2(I) = 6$, $a_3(I) = 2$. We get that

$$\begin{aligned} d(I; n) &= 5 \binom{n-3}{1} + 6 \binom{n-3}{2} + 2 \binom{n-3}{3} \\ &= \frac{1}{3} (3 - n - 3n^2 + n^3). \end{aligned}$$

In total, there are

$$d(I; 6) = \frac{1}{3} (3 - 6 - 3 \cdot 6^2 + 6^3) = 35$$

permutations in S_6 with descent set equal to $\{1, 3\}$.

At first, this formula is mysterious, but reading the proof in [5] reveals the key idea is that the number of permutations with specific descents stabilizes, up to replacing certain values in the permutation with a choice of larger ones. This choice is precisely what leads to the binomial coefficients in the formula. To understand what this means, let's look at the example above again.

Example 7 Once again, consider $I = \{1, 3\}$. This time let's consider $d(I; 8)$, the number of permutations in S_8 with descent set I . Our formula from Example 6 tells us that we should get

$$\frac{1}{3} (3 - 8 - 3 \cdot 8^2 + 8^3) = 105.$$

The key insight is to group permutations by how many of the numbers $\{4, 5, 6, 7, 8\}$ appear within the first three entries. For example, here are all

the permutations in S_8 with the descent set $\{1, 3\}$ and such that only one of the numbers 4, 5, ..., 8 appears in the first three entries.

$$\begin{array}{cccccc} 21\textcolor{red}{4}35678 & 21\textcolor{red}{5}34678 & 21\textcolor{red}{6}34578 & 21\textcolor{red}{7}34568 & 21\textcolor{red}{8}34567 \\ 31\textcolor{red}{4}25678 & 31\textcolor{red}{5}24678 & 31\textcolor{red}{6}24578 & 31\textcolor{red}{7}24568 & 31\textcolor{red}{8}24567 \\ 32\textcolor{red}{4}15678 & 32\textcolor{red}{5}14678 & 32\textcolor{red}{6}14578 & 32\textcolor{red}{7}14568 & 32\textcolor{red}{8}14567 \\ \textcolor{red}{4}1325678 & \textcolor{red}{5}1324678 & \textcolor{red}{6}1324578 & \textcolor{red}{7}1324568 & \textcolor{red}{8}1324567 \\ \textcolor{red}{4}2315678 & \textcolor{red}{5}2314678 & \textcolor{red}{6}2314578 & \textcolor{red}{7}2314568 & \textcolor{red}{8}2314567 \end{array}$$

We see that permutations in which 4 is the only number greater than 3 that appears in the first three entries are in bijection with the permutations where 5 is the only number greater than 3 that appears in the first three entries, and similarly for 6, 7, and 8. We also see that each of these sets are in bijection with the permutations appearing in (2). A similar pattern appears for the permutations that contain two or three of the numbers $\{4, 5, 6, 7, 8\}$ in the first three entries.

A Generalization of Descent Polynomials

We can view the descent set of a permutation $w \in S_n$ as a subset of the inversion set of w . Indeed,

$$i \in \text{Des}(w) \Leftrightarrow (i, i+1) \in \text{Inv}(w) \quad (3)$$

so the set of descents is in natural bijection with the set $\text{Inv}(w) \cap \{(i, i+1) \mid i \in \mathbb{N}\}$. When we view descents in this light, there is a natural generalization—can we count permutations with particular inversions instead? The answer is yes, and the rest of this article explores some joint work between myself, Jeongwon Lee, and Matthew Giardinelli.

Define

$$\text{Inv}_2(w) := \{(i, j) \in \text{Inv}(w) \mid j - i \leq 2\},$$

to be the set of inversions of w that occur at most two positions apart.

Example 8 If $w = 36281475$ as in Example 2, then

$$\text{Inv}_2(w) = \{(1, 3), (2, 3), (3, 5), (4, 5), (4, 6), (7, 8)\}.$$

Let $K \subset \{(i, j) \mid i < j, j - i \leq 2\}$ and let's count the number of permutations in S_n such that $\text{Inv}_2(w) = K$. Notice that this count refines the count of permutations with a specific descent set since descents correspond to consecutive inversions using (3). Define

$$d(K; n) = \#\{w \in S_n \mid \text{Inv}_2(w) = K\}.$$

As before, the set will be empty unless n is large enough. Let's try some examples.

Example 9 Let $K = \{(1, 2), (3, 4)\}$. Then K is the analogue of the descent set $\{1, 3\}$ as considered in Examples 6 and 7, identified as a set of inversions using (3). Here are the permutations in S_4 , S_5 , S_6 , and S_7 , respectively, such that $\text{Inv}_2(w) = K$:

$$\begin{aligned} 2143, 3142 &\in S_4, \\ 21435, 31425 &\in S_5, \\ 214356, 314256 &\in S_6, \text{ and} \\ 2143567, 3142567 &\in S_7. \end{aligned}$$

In each case, there are only two! In fact, we have $d_2(\{(1, 2), (3, 4)\}; n) = 2$ for all n .

Example 10 Let $K = \{(1, 2), (1, 3)\}$. Let $n \geq 3$ and consider a permutation $w \in S_n$ with $\text{Inv}_2(w) = K$. Then

$$\begin{aligned} w(1) &> w(2), w(1) > w(3), \text{ and} \\ w(2) &< w(3) < \cdots < w(n). \end{aligned}$$

Consider the positions of 1 and 2 in the one-line notation of w . The conditions $w(1) > w(2)$ and $w(1) > w(3)$ tell us that $w(1) > 2$. Since the entries $w(2), w(3), \dots, w(n)$ must be increasing we conclude that $w(2) = 1$ and $w(3) = 2$. We can choose $w(1)$ to be any of $3, \dots, n$. Thus, $\text{Inv}_2(w) = n - 2$.

It is possible to give a formula for $d_2(K; n)$ that is similar to the formula appearing in Theorem 2 for the descent polynomial $d(I; n)$. In particular,

Theorem 3 (Giardinelli, Lee, Precup). *For all*

$$K \subset \{(i, j) \mid i < j, j - i \leq 2\},$$

$d_2(k; n)$ is a polynomial in n with nonnegative integer coefficients in the (shifted) binomial basis.

An interesting difference between $d_2(K; n)$ and the descent polynomial is that $d_2(K; n)$ can be zero for certain subsets K , and our results include a characterization of when this occurs. Our work on the polynomial $d_2(K; n)$ is ongoing. There are many generalizations and connections to other combinatorial objects.

- Pakawut and McConville give an interpretation of the descent polynomials as enumerating standard Young tableaux of a ribbon shape and use it to study the roots of the polynomial [7].
- One can study the number $p(I; n)$ of permutations in S_n with a given set of peaks. This expression is not a polynomial, but one can prove that it is of the form $2^n q(I; n)$ where $q(I; n)$ is a polynomial called the *peak polynomial* [2, 4].

- Gaetz and Gao defined a refinement of both descent and peak polynomials by adding a variable to keep track of the length of the permutation [6].

These are only a few! There is always more to explore.

References

- [1] Ferenc Bencs. “Some coefficient sequences related to the descent polynomial”. In: *European J. Combin.* 98 (2021), Paper No. 103396, 20. URL: <https://doi.org/10.1016/j.ejc.2021.103396>.
- [2] Sara Billey, Krzysztof Burdzy, and Bruce E. Sagan. “Permutations with given peak set”. In: *J. Integer Seq.* 16.6 (2013), Article 13.6.1, 18.
- [3] Petter Brändén. “Unimodality, log-concavity, real-rootedness and beyond”. In: *Handbook of enumerative combinatorics*. Discrete Math. Appl. (Boca Raton). CRC Press, Boca Raton, FL, 2015, pp. 437–483.
- [4] Alexander Diaz-Lopez et al. “A proof of the peak polynomial positivity conjecture”. In: *J. Combin. Theory Ser. A* 149 (2017), pp. 21–29. URL: <https://doi.org/10.1016/j.jcta.2017.01.004>.
- [5] Alexander Diaz-Lopez et al. “Descent polynomials”. In: *Discrete Math.* 342.6 (2019), pp. 1674–1686. URL: <https://doi.org/10.1016/j.disc.2019.01.034>.
- [6] Christian Gaetz and Yibo Gao. “On q -analogs of descent and peak polynomials”. In: *European J. Combin.* 97 (2021), Paper No. 103397, 11. URL: <https://doi.org/10.1016/j.ejc.2021.103397>.
- [7] Pakawut Jiradilok and Thomas McConville. “Roots of descent polynomials and an algebraic inequality on hook lengths”. In: *Electron. J. Combin.* 30.4 (2023), Paper No. 4.41, 32. URL: <https://doi.org/10.37236/10753>.
- [8] Percy A. MacMahon. *Combinatory analysis*. Two volumes (bound as one). Chelsea Publishing Co., New York, 1960.

Mentoring Math Minds Program Participants Testimonials

In the Fall of 2024, the Department of Mathematics and Statistics at Saint Louis University launched a Research Mentoring Program designed to support both graduate and undergraduate students. Led by Dr. Ghoreishi, Dr. Pozzi, and Dr. Uğurlu, the program provides opportunities for students to engage in mathematical research, develop professional skills, and build a supportive math community. In their own words, participants reflect on how this program has shaped their journeys.

Jehan Alarfaj

Being part of the Mentoring Math Minds Program has provided me with invaluable guidance. It has allowed me to engage in meaningful academic discussions, receive constructive feedback, and receive support in refining my research and presentations. The opportunity to connect with peers, learn from their experiences, and explore various conference opportunities has been truly enriching.

What makes this program even more special is the genuine willingness of the team to help, creating an environment where students feel comfortable asking questions and reaching out for support. It's not like a traditional classroom setting—it's a collaborative space that encourages growth, open discussion, and mentorship in a way that feels both supportive and inspiring.

Sam Doll

While attending Saint Louis University, I have found that the most influential and inspiring part of my education has been participating in the Mentoring Math Minds Program. I transferred to Saint Louis University not knowing how I wanted to advance my mathematical ventures, but being part of the program has opened my eyes to countless endeavors I could pursue with my degree. Being directly connected with knowledgeable professors and peers

has elevated my understanding of mathematical research and connected me with numerous networking and research opportunities. The program has provided me with greater connections to potential colleagues, peers, and professors who share a deep connection with mathematics and encourage me to think deeply about what I want out of my mathematics program and degree. The workshops have engrossed me in technology that I may not have had the chance to use in the classroom, such as LaTeX/Overleaf, and have given me the opportunity to showcase my interpretations of mathematics. The program has left me with greater confidence in my ability, a greater sense of mathematical community, and a deeper connection with Saint Louis University as a whole.

Preetinder Kaur

I joined the Mentoring Math Minds Program in the Fall of 2024 as one of the first scholars. Initially, I had some expectations, such as discussing research with the mentors. However, since this was the program's inaugural semester, the experience was new for both the faculty and the participants, so I did not have many preconceived notions beyond that. Throughout the program, my expectations were not only met but exceeded.

One of the most valuable aspects of the program was the opportunity to engage in one-on-one interactions with visiting mathematics faculty and colloquium speakers. From a student's perspective, the journeys of these professors—how they explored their careers and defined their goals—often seemed distant and unattainable. However, these conversations provided a window into their experiences, making a career in mathematics, or even pursuing a graduate degree, feel within reach.

Another significant component of the Mentoring Math Minds Program was the series of workshops designed to enrich our professional and academic

experiences. Attending a LaTeX workshop was particularly beneficial for me. Before this, I had never even heard of LaTeX, but the introduction to professional mathematical writing was invaluable. As someone taking a statistics course with R, I found that incorporating LaTeX into my assignments greatly improved their presentation. Additionally, the resume-building workshop was incredibly useful. Although resume-building is occasionally discussed in core classes, this workshop provided an opportunity to ask more personalized questions. Moreover, the session on personal statements was eye-opening. While I had heard of personal statements before, I had never received guidance on what differentiates a well-written statement from a truly captivating one. Learning how to make a personal statement reflective of my personality and traits while staying relevant to the topic was a game-changer.

Through this program, I was also introduced to the Nebraska Conference for Undergraduate Women in Mathematics (NCUWM). Before my mentors mentioned it, I had never heard of this conference and likely never would have. Attending NCUWM was a transformative experience. The road trip to Nebraska allowed me to bond with two of my three mentors in a way that transcended the traditional student-professor dynamic—we were simply people excited to explore the conference together. (Not to say that professors aren't people outside of class, of course!)

My first reaction upon attending the conference's welcome orientation was amazement at the sheer number of undergraduate women majoring in mathematics. The realization that this was just a fraction of the field was even more astonishing. The conference was likely the only place where I would ever see so many female math professors gathered in one location. Hearing about their experiences challenges they faced and the successes they achieved was truly inspiring.

One aspect that stood out to me in both the Mentoring Math Minds Program and the conference was the exposure to career paths beyond academia. While graduate degrees in mathematics were frequently discussed, there were also sessions on industry applications of mathematics, which was refreshing. Before this, the only people I had met with mathematics degrees were professors. As someone majoring in both Computer Science and

Mathematics, I greatly appreciated this broadened perspective, which reaffirmed my decision to pursue a math major.

I wouldn't go so far as to say that the Mentoring Math Minds Program and NCUWM completely changed my perspective on mathematics, as I have always loved the subject. However, they certainly opened my eyes to the vastness of the field and the numerous opportunities within it. These experiences deepened my appreciation for mathematics and expanded my understanding of the many directions my studies could take me.

Sam Liu

If you're even slightly curious about what a career in mathematics, education, or academic research looks like, apply to the Mentoring Math Minds Program. Dr. Pozzi, Dr. Uğurlu, and Dr. Ghoreishi have an unbelievable knack for mentoring students and an immeasurable dedication to the cohort, which I know to be exceptional. I recall that in the first few weeks of the program, an eminent researcher came to the university to give a colloquium talk, and the mentors arranged for us to have a private exchange with him. Arriving early, I found Dr. Pozzi already engaged in conversation with the visiting scholar. She quickly introduced me to him, and her enthusiasm was palpable—the way she spoke about me made it clear that I was a part of the great mathematical research conducted in the department. Dr. Pozzi has long supported my ambitions even before the program, but that moment during the program made me realize that I belonged in the Saint Louis University math community. Later, during the conversation, Dr. Uğurlu asked the scholar a question I'll never forget: "So, do you have any research opportunities for our students?" That's who Dr. Uğurlu is—a reliable, trustworthy person who is always thinking of my best interests.

As you work through the program, you will attend a series of workshops designed to fundamentally grow your professional and academic potential and create presentations on topics that excite you. This semester, the students are tasked with assembling and giving a poster presentation. Since last November, I've been working with Dr. Ghoreishi to solidify a geometric understanding of linear algebra and to learn about its applications in frame theory. Often, I find studying mathematics and writing proofs chal-

lenging to start. Dr. Ghoreishi has been incredible in meeting me where I am and challenging me to foster good learning practices, which has completely changed my perspective on approaching mathematics. Dr. Ghoreishi's kind and patient love for mathematics is infectious and has ignited a passion in the deepest parts of my consciousness. My experience as a 2024–25 mentee has been fulfilling, to say the least. This past year, I've grown by creating memorable, meaningful experiences: learning technical skills, building projects, engaging in professional development, and connecting with a greater community. These three important people have given my undergraduate education its rigorous shape. To Dr. Pozzi, Dr. Uğurlu, and Dr. Ghoreishi, thank you for investing in me. You are a fundamental part of my undergraduate experience, and this junior year would have been incomplete without you. To all prospective students: just apply—you will not regret it.

Nat Stonebrook

I joined this program at an interesting time in my life. At the age of 28, I had been out of school for over 10 years, which meant that I had gone a very long time without any mathematical training. When I applied to the program, I had felt an insecurity that this gap would make it hard for me to participate, or worse, that I would not be allowed in. But without much waiting, I received an email asking me to meet with each of the mentors (Dr. Ghoreishi, Dr. Pozzi, and Dr. Uğurlu). In moments of my first meeting, I was offered assurance that this academic gap would have no bearing on my time with the program, and this assurance was repeated in the following meetings.

Soon after joining, I began working with a mentor on a research project. When I entered her office for the first time, I observed the blackboard and saw a scattering of equations, partial derivatives, and symbols for functions that weren't an f or g . We started working immediately. She wanted to catch me up on her projects and some of the notation, but when she got to a particular sum, she introduced a symbol and turned to me: "Is this okay?" she said, gesturing to the board. I nodded in agreement. The symbol in question was the Greek capital letter sigma, a notation which is learned long before the partial derivatives surrounding it, and to ask me if it was known to me showed a compassion that has been shared by my other mentors. It is

never easy for professional mathematicians to place themselves in a beginner's mind, but they have made it seem very effortless.

These things and many others put my mind at ease, and I was able to attend the weekly meetings, ask questions, and voice myself, without any concern for looking silly. I am now in my 2nd semester with Mentoring Math Minds, and I have met with dozens of professors from around the world. Each week, or nearly every week, we get the opportunity to sit down with a visiting professor and ask them anything we'd like. For professional reasons, this has the obvious benefit of gaining contacts in a very competitive field; and for developmental reasons, one gets to hear from someone in the field talk about what it's like working in academia. As well, we've had workshops on LaTeX (which I now use nearly every day), professional skills like resume building, and more academic things like mathematical analysis.

I am deep into two reading courses, working on an expository paper and poster, and soon participating in another workshop for Python. These are skills that are becoming, or already are, necessary to be a working mathematician, but many of them are not taught to students until they need them. Learning them early, having access to office hours with the mentors, and getting the chance to type up real documents are invaluable.

Mentoring Math Minds is an oddly unique program, and having a sense of belonging in the mathematics community is a wonderful thing. Joining has been one of the better decisions of my life, and I'm looking forward to coming back one day as a mathematician myself.

Sasha Trejo-Arciles

As someone who discovered their passion for mathematics in their final year of college, I realized how much I needed mentorship. The prospect of attending graduate school felt incredibly daunting and uncertain. Additionally, while I knew I wanted to pursue something involving mathematics after graduation, I wasn't exactly sure what that path would look like. Through the Mentoring Math Minds Program, I have gained experiences that have been profoundly transformative in both my personal and professional life. The mentors in the program have been incredibly supportive and have

provided guidance every step of the way. Their encouragement, along with the program's overall welcoming and supportive environment, has helped me navigate my goals with greater clarity and confidence. I now feel much more confident about my future in mathematics.

Inquiries about the program?

Contact us at: mentoring.math@slu.edu



Visit us on mentoringmathmind.wordpress.com

What is Happening in Ritter?

Association for Women in Mathematics Student Chapter

The SLU Student Chapter of AWM mentors and encourages women and other minorities as they prepare for careers in mathematical sciences. We encourage conversation in the mathematics community among women and other gender minorities. This includes connecting undergraduate students, graduate students, and faculty at both formal talks and casual mathematics gatherings. While this is a math chapter, it is not only aimed at math students but rather at all neighboring fields and interests. We meet on Mondays at 4:00 pm. Check out the events and more information at the [department's website](#) or through [SLU Groups page](#).



Math Club

The Math/Stats club meets on Wednesdays at 4 pm in the Ritter Hall Lobby. Events include outside speakers, games, competitions, field trips, and other opportunities to socialize with fellow students. Students of any level and faculty are all welcome to attend club events. Find more information about the club on the [SLU Groups page](#).



Colloquium

The department runs weekly colloquia featuring speakers from both SLU and other top national institutions. The events are categorized into researched-focused and education-focused talks. The departmental colloquium meets on Fridays at 4:00 pm with refreshments beforehand in the Ritter Hall Lobby. Check out [department's events website](#) to see the calendar for all departmental events.



Math in the Wild Challenge

Dr. Bryan Clair

That's me, thinking about L'Hôpital's rule. However, I found out later that this is the wrong L'Hôpital!



From Versailles, France (quite some time ago)

Dr. Russell Blyth

During my sabbatical in 2022; taking long walks along the Arno and came across these graffiti under the train bridge racks.



From Arno in Florence, Italy

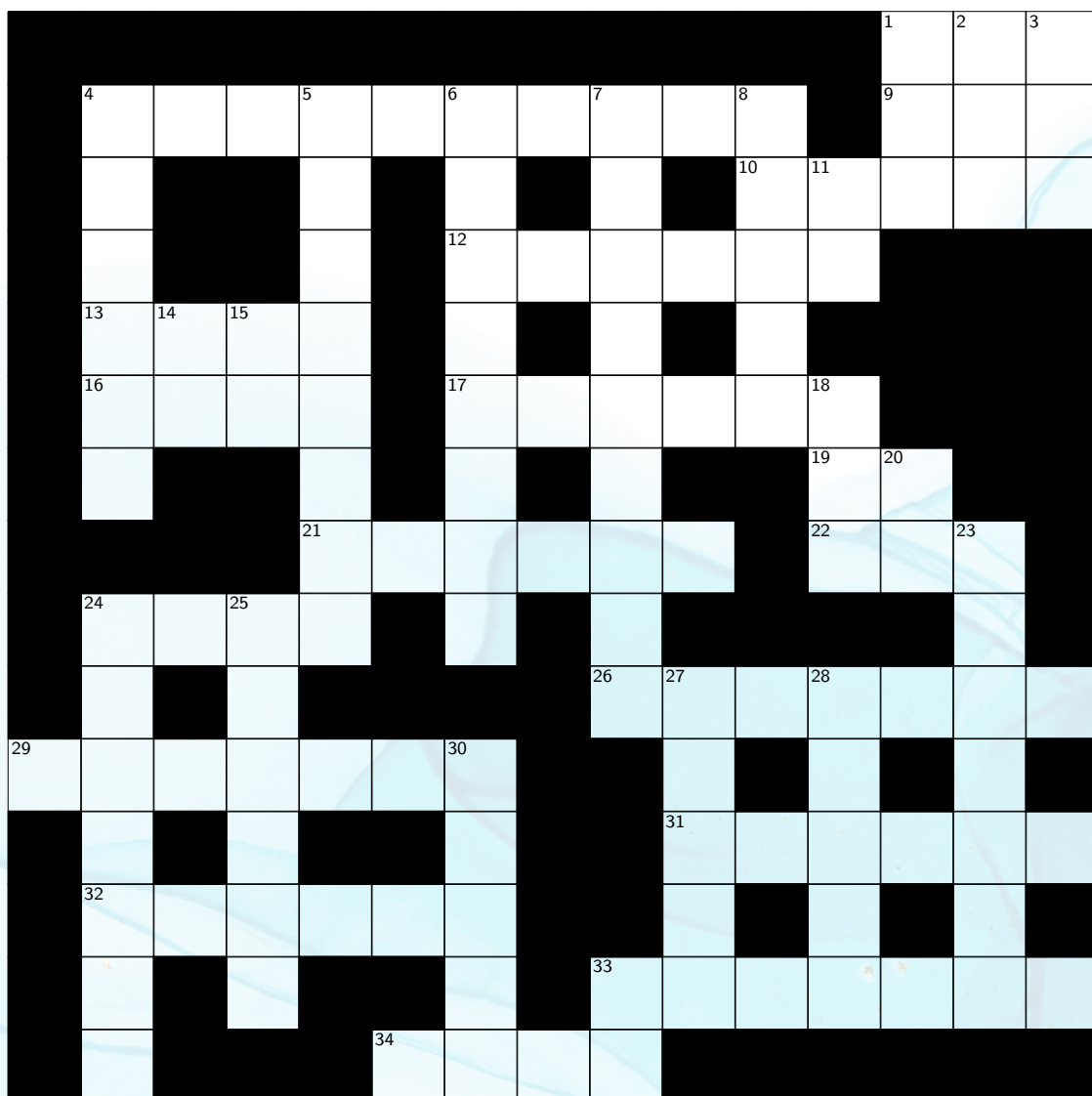
Dr. Michael Landry

My collaborator Steven Frankel and I are working together on defining new surgery operations on quasi-geodesic pseudo-Anosov flows. Last summer he received some great tickets to a Cardinals game, but the timing conflicted with our regular meeting. The solution we came up with was to hold our meeting at the game. Steven took this picture, which shows me watching the game, as well as my notebook in the foreground.



Busch Stadium, Saint Louis, United States

Crossword Puzzle
Created By Kieran Favazza



Across

- 1 The test a senior dreads when applying to graduate school
- 4 Famously said "Eureka!"
- 9 Research institution famously home to Einstein and Oppenheimer
- 10 Famous Hungarian analyst whose name graces an MAA prize
- 12 2 or e , for example
- 13 Injective, colloquially
- 16 Kernel of a linear operator: the ___ space
- 17 Not infinite
- 19 Comes after 14 down
- 21 French mathematician that famously died in a duel
- 22 With 18 down, makes the fundamental tools in homological algebra
- 24 Institute that awards the Millennium Prize
- 26 Influential female mathematician whose name graces isomorphism theorems
- 29 In $\mathbb{R}^n$, closed and bounded
- 31 Names a kind of cohomology for manifolds
- 32 A space admitting a free and transitive action of a group
- 33 What one might do at a conference
- 34 Prominent Italian mathematician whose namesake subgroups are foundational in Lie theory

Down

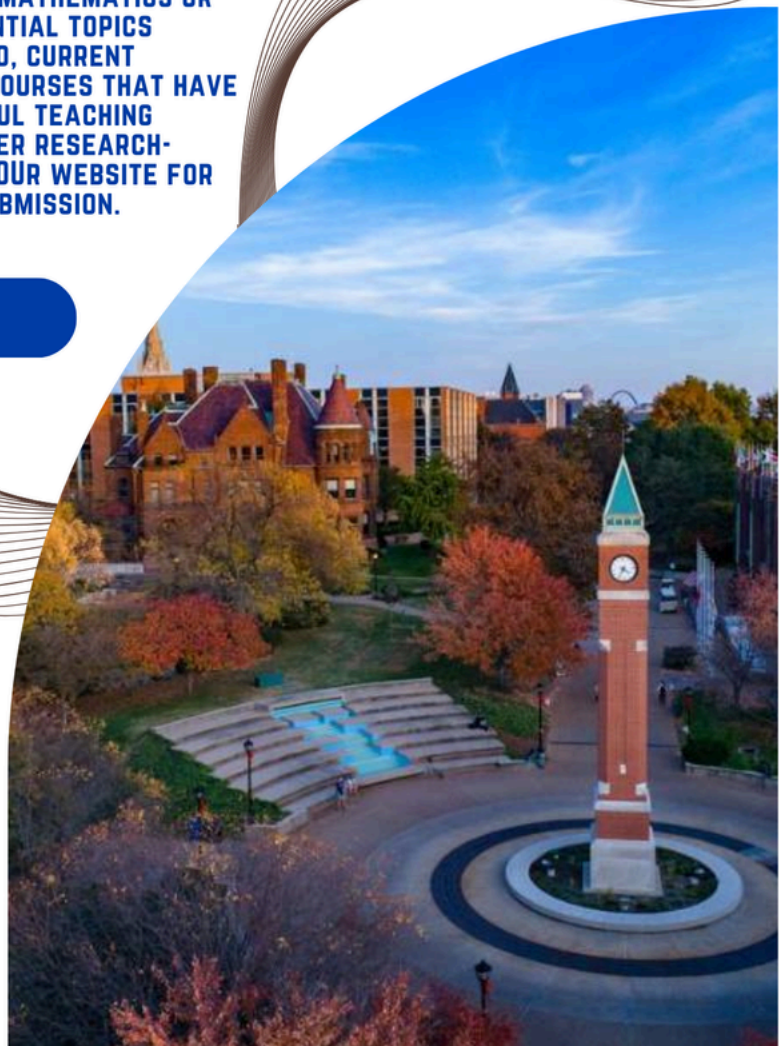
- 1 First name of the famous linear algebra educator Strang
- 2 A half-line
- 3 European counterpart to NASA
- 4 Flat; describes hyperplanes not passing through the origin
- 5 Quotient of kernel by image
- 6 A locally flat space
- 7 Cardinality of a basis
- 8 The collection of all prime ideals of a ring, abbr.
- 11 With "AND" and "NOT," makes up the basic operators in logic
- 14 Comes after μ
- 15 Initials of the Italian mathematician that names a connection on manifolds
- 18 With 22 across, makes the fundamental tools in homological algebra
- 20 One of Galileo's moons
- 23 Student of Gauss
- 24 (Opted)*, or what L'Hôpital did to a rule discovered by Bernoulli
- 25 SI unit of electric current
- 27 In a group, the smallest integer m such that $g^m = 1$
- 28 What one might call the summands in a series
- 30 $\lfloor \pi \rfloor$
- 33 π

MAA MISSOURI SECTION MEETING 2025

AT SAINT LOUIS UNIVERSITY
APRIL 4-5, 2025

WE INVITE BOTH FACULTY AND STUDENTS TO PRESENT AT THE SPRING MEETING. PRESENTATIONS SHOULD BE 15 MINUTES LONG AND FOCUS ON MATHEMATICS OR MATHEMATICS EDUCATION. POTENTIAL TOPICS INCLUDE, BUT ARE NOT LIMITED TO, CURRENT MATHEMATICS RESEARCH, NEW COURSES THAT HAVE BEEN DEVELOPED AND SUCCESSFUL TEACHING TECHNIQUES. TALKS CAN BE EITHER RESEARCH-ORIENTED OR EXPOSITORY. VISIT OUR WEBSITE FOR REGISTRATION AND ABSTRACT SUBMISSION.

REGISTER HERE!



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