

FOR ALL

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SCAN ME

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Saint Louis is our home, and we endeavor to make it a better place by creating inclusive communities that can share in the joy of learning and the city's beauty.

ABOUT US

Biannual inclusive mathematics magazine featuring articles and expositions on a wide range of topics related to mathematics and more.

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Welcome to *For∇ll*

Welcome to another exciting edition of our math magazine For∇ll! We are thrilled to have you on board as we explore a wide range of mathematical perspectives and discoveries. In this issue, we aim to make the world of math accessible to all, regardless of your background. Whether you are an experienced mathematician or just starting your journey, our goal is to foster a sense of inclusivity and curiosity. We encourage you to explore, question, and engage with the content within these pages.

Moreover, we extend a warm invitation for submissions from all mathematicians - undergraduates, graduates, and experienced researchers alike. Your contributions add depth and variety to our journal, and we particularly encourage students to share their unique insights. Whether it's an undergraduate project or a groundbreaking research endeavor, we value the richness each perspective brings to our mathematical community.

Thank you for being a part of our journey, and we hope you find inspiration and enjoyment in the pages that follow.

Follow us and submit your content on our website!



This is a journal of the people in our community, and it is strengthened by any art, puzzles, or articles the community can provide.

For∇ll Staff and Editorial Board

- Editor in Chief - Özlem Uğurlu, Assistant Professor at Saint Louis Univerity
- Student Editor - Kieran Favazza, Undergraduate student at Saint Louis Univerity

Faculty Interview Series

Interviewing: Bahareh Rahmani

Interviewed by: Kieran Favazza



Introduction

Dr. Bahareh Rahmani joined the Saint Louis University Department of Computer Science in the Fall of 2022 and has already been involved in a large swath of classes. She earned her Ph.D. in computer science - bioinformatics from the University of Tulsa in 2016 after completing her bachelor's degree at Sharif University in Tehran, Iran, and she has developed and taught over 25 different courses since the completion of her Ph.D. She loves teaching and research alike - she loves to see how her work can help others and offer something useful in their lives. She offers her unique perspectives on mathematics as our first non-math faculty member in the faculty interview series, and we are excited to hear her insights!

Interview

Dr. Rahmani, how did you become interested in computer science?

I actually got my master's in applied math, so I have always been a little more mathematical, but I wanted to get my Ph.D. in computer science so that I could see the application of math somewhere and be involved with it. I really like math, but I found the pure math side of things to be a little too far removed from reality and hence harder on the brain.

How did your upbringing influence you to like math so much?

Growing up in Iran, it was always a goal of mine (and of many) to go to the best university in the country, and it so happened that this university was technology-centered - mostly focused on engineering and math careers. I was always good at math, and I found electrical engineering and computer engineering to be quite difficult and not the most fun, so I chose to study math. I enjoyed this and hoped to transition to something a little more focused on engineering eventually, but at that time, it wasn't possible in my country to go from math to computer science. That was when I decided to come to the United States and get my Ph.D. in computer science - bioinformatics from the University of Tulsa.

Was it challenging to be in that position as an undergraduate?

It was very challenging. The students at my undergraduate university were the ones with the best grades in the whole country, so the competition with them alone was quite challenging. But it was still so much fun because I enjoyed the math so much, so it was definitely worth it.

Do you enjoy math more for the problem-solving aspects or more for the practice of scientific discovery?

I definitely enjoy math the most for its applications. I am teaching a machine learning class right now, and it is really the type of thing that I enjoy. Another faculty member in the department is teaching a class on the mathematical foundations of machine learning, but I am teaching a more applied version, in which we work with a lot of data and apply different machine learning algorithms to visualize and capture a lot about these interesting data sets. My

master's thesis was where I got into this type of practice; we used some interesting MATLAB programs to analyze how computers can help in the optimal design of automobiles.

You started with computer-aided design but ended up getting your Ph.D. in bioinformatics. What has been your favorite vein of computer science/applied math?

I would say that I am more like a data scientist now. I like to extract information and communicate that to professionals to find the best solutions. I generally like working with medical data a lot because I can really see the results - I like seeing that my work is useful for someone and can help people. We have recently been working on some apps and websites to which healthcare providers can upload X-rays or lab data that will automatically interpret the data and return certain diagnoses.

You have taught over 25 different courses in the past 8 years. What has been your favorite to teach?

My favorite course to teach is computer vision. It is fun to talk about, and I enjoy project-based classes. Also, when I was at Fontbonne University, I designed 7 courses from scratch for data science, and I loved doing that. It was nice to build up these things that seem essential for any data scientist and contribute to the next generation of students.

What are you currently working on in your research?

Right now, I am writing a lot of grant proposals, but we have been working to find optimal models that can assess radiological data and make accurate diagnoses. We haven't been creating new algorithms per se, but we have been fitting different models to our data, adjusting them, and combining them accordingly with our other models and goals for the project to develop this app that healthcare providers can use to make quick diagnoses based on complex laboratory data. Hopefully, once these grant proposals are accepted, we can bring in doctoral students or postdoctoral scholars to develop new, even better models for our app. For example, one of the capstone projects I am supervising is developing models specifically for analyzing data on the lungs - determining whether certain data indicate various respiratory diseases or needs for specific treatments.

What has been your experience as a woman in STEM? Can you share any advice for young women in STEM?

It can of course depend on your personal life, but in Iran, I had a great experience. There are more women than men in universities in Iran, so I was given a lot of support as a student. My family supported me, and I was treated with respect when I came to the US. I was supported as a graduate student, and I found no issues with my gender in being hired at universities. Personally, I feel that universities are the best places for women to grow; especially international women. My advice to younger people and students who experience prejudices - especially young women who don't receive adequate support in their studies - is to find mentors who will help. This is why it is

important to have diverse faculty at a university; I have heard from many female students that they find it easier to approach female instructors for help, and I am glad that I have been able to assist students in this regard. Certain students of mine have experienced prejudices or difficult lives outside of school, and I was able to help them by listening, being kind, and trying to understand how I can best support them and their situation.

What do you like to do outside of your work? Do you find it hard to balance work and life?

I have two children, so they of course keep me busy, which I love. I get the work done that I must do during the year, but I love teaching because it allows me more time with my family over the summer. I usually use my summer to go home and visit family in Iran, which is always nice. I don't have time for much else, but I like my work and love keeping up with everything for my kids.

Conclusion

Dr. Rahmani is a great example of how math can pervade our lives. Often, if we work more on the theoretical side of things, we can get caught up in our abstract ideas and forget how our work affects the real world. Dr. Rahmani focuses her mathematical efforts on the concrete issues that affect people around us and serves as a reminder that we should all appreciate the teachers, researchers, and mathematics that shape our daily lives.

MATH

$y = mx + b$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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$V = s^3$

$V = Lwh$

$V = \frac{4}{3}\pi r^3$

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$

$C = 2\pi r$

A Journey through My Undergraduate Research

By Aksheytha Chelikavada

Undergraduate Student, Department of Mathematics and Statistics, Saint Louis University

I have a deep interest in mathematics and statistics and find joy in the challenges they present. Additionally, I recognize the pivotal role they play in driving innovation and advancement. So, beyond coursework and class projects, I wanted to do something more impactful under the guidance of a research professor. After discussing my interests with Dr. Clair, I was recommended to Dr. Panzo as a research assistant. Following some insightful conversations, Dr. Panzo and I decided to work on limit theorems for fixed point biased permutations avoiding a pattern of length three. This project combined our interests in probability theory, combinatorics, and computer science.

For the project we worked on, we studied the impact of bias on pattern-avoiding permutations with fixed points. Permutations are an arrangement of data elements, and most real-world datasets have some level of inherent patterns. Fixed points help us measure the degree of sortedness in permutations. Certain patterns were avoided, and bias was applied to lead to a specific arrangement of elements. While utilizing Markov chain simulation, as the length of the permutation was increased, the number of fixed points started to exhibit a behavior that aligns with the properties of a probability distribution. With these results, we can predict future trends more accurately when patterns may not exist.

Being a part of this research project was truly an intellectual adventure. Initially, I started reading literature to understand the current discoveries and advancements in probability theory and combinatorics. Though it was challenging, Dr. Panzo's guidance aided me in having a more nuanced understanding of them. We started to identify gaps or areas for further investigation and formulated a methodology to address our questions. In the next phase, we utilized statistical analysis to extract meaningful conclusions.

Seeking answers and uncovering new information during this project proved to be intellectually stimulating and satisfying. This project not only encouraged me

to think critically but also nurtured my ability to solve challenging problems. Furthermore, I enjoyed the interpersonal aspect of working closely with Dr. Panzo applying knowledge in Mathematica, Python, R, and LaTeX towards a common objective.

At the moment, we are in the process of publishing our work. Learning about academic publishing has been very informative. I have learned about journal selection, manuscript formats, peer reviews, and publication standards through the process. And I must say, by going through the publication process, I have gained an appreciation for scientific publishing.

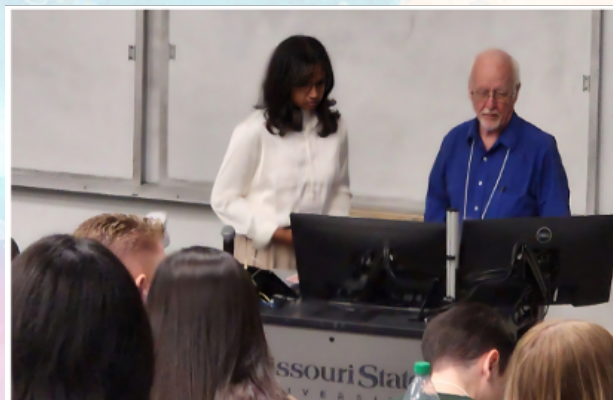


Figure 1: At the MAKO conference that took place at Missouri State University, Dr. Les Reid, who is the organizer of the conference and a professor at the university, helped me get my materials ready for my presentation.

While being a part of this publication journey, I had the privilege to showcase our research work at the MAKO (Missouri, Arkansas, Kansas, and Oklahoma) conference (see Figure 1), Joint Mathematics Meetings (see Figure 2), and Nebraska Conference for Undergraduate Women in Mathematics, all of which gathered good interest and questions. These opportunities allowed me to share findings and connect with the academic community. At the same time, I was able to stay updated on the latest developments in the field. I am deeply grateful

to Dr. Panzo and Dr. Clair for generously sharing these invaluable opportunities and for their unwavering support and encouragement throughout this journey.

Looking ahead, I am eager to continue my academic journey fueled by the experiences I gained during my time as a research assistant. I would like to build on what I have learned, advance my skills, and explore new questions.



Figure 2: At the Joint Mathematics Meetings in San Francisco, I was preparing for my presentation.

Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called “EvenQuads” to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.



Moon Duchin
(b. late 1970s)

Moon Duchin earned a mathematics PhD from the University of Chicago (2005) and is currently Associate Professor of Mathematics at Tufts University, a founder of their Science, Technology, and Society Program, and leads the Metric Geometry and Gerrymandering Group (MGGG) Redistricting lab. Duchin was a principal co-author of an Amicus Brief cited by the US Supreme Court and a consulting expert for congressional redistricting efforts in Pennsylvania. Duchin received an NSF CAREER grant (2013–2018), is an AMS Fellow (2017), Guggenheim Fellow (2018), and Radcliffe Fellow (2018), has served on the board of Spectra (2017–2022), and served on the AMS committees for human rights of mathematicians (2016–2019) and science policy (2020–2023).

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.

EVENQUADS



What is Happening in Ritter?

Association for Women in Mathematics Student Chapter

The SLU Student Chapter of AWM mentors and encourages women and other minorities as they prepare for careers in mathematical sciences. We encourage conversation in the mathematics community among women and other gender minorities. This includes connecting undergraduate students, graduate students, and faculty at both formal talks and casual mathematics gatherings. While this is a math chapter, it is not only aimed at math students but rather at all neighboring fields and interests. We meet on Mondays at 4:00 pm. Check out the events and more information at the [department's website](#) or through [SLU Groups page](#).



Math Club

The Math/CS club meets on Wednesdays at 4 pm in the Ritter Hall Lobby. Events include outside speakers, games, competitions, field trips, and other opportunities to socialize with fellow students. Undergraduate students, graduate students, and faculty are all welcome to attend club events. Find more information about the club on the [SLU Groups page](#).



Colloquium

The department runs weekly colloquia featuring speakers from both SLU and other top national institutions. The events are categorized into researched-focused and education-focused talks. The departmental colloquium meets on Fridays at 4:00 pm with refreshments beforehand in the Ritter Hall Lobby. Check out [department's events website](#) to see the calendar for all departmental events.



From Chemistry to Math: My Story

By Madi Sousa

Doctoral Student, Department of Mathematics, Vanderbilt University

Part I: My Math Journey and Women Mentors

I could say that I instantly fell in love with mathematics as a child; that multiplication or the Pythagorean theorem inspired me to become a mathematician. But that would not be true. In fact, I did not like math at all throughout high school, and I had to force myself to register for a second calculus class after hating the first course while in college. However, I knew that I needed a mathematical background for my major at the time, namely, chemistry.

I deeply believed that I wanted to be a chemist because I was fascinated by physical interpretations of the natural world. To further my knowledge in chemistry, I was involved in multiple research groups across two universities focusing on the optical properties of nanomaterials. I quickly learned that to understand the experiments that I ran and the theory behind the physics of molecular interactions, I simply needed to know more math. With that, I began to take classes to earn a second degree in mathematics.

To my surprise, the math classes that I was taking at the time were far more interesting than any of the chemistry I had learned or researched. I spoke with my mathematics professor at the time, asking for his advice on whether I should drop my chemistry major to dedicate myself to learning mathematics. In response, he told me that I did not have the potential to be a mathematician. This would not be the only time in my early mathematics career that I was told to give up my interest in the subject, mainly due to the fact that I was a woman. In total, I would have three mentors in my undergraduate career who believed mathematics was meant primarily for men.

I knew that I could not let go of my newfound interest in mathematics; consequently, during my third year of college, I applied for the Association for Women in Mathematics (AWM) Mentor Network Program to be paired with a professional mathematician to hopefully find an outlet to explore mathematics outside of my university. More specifically, I requested a mentor to teach me the theory behind phase retrieval in optical

imaging, as this was an integral part of my research in chemistry. Little did I know that this connection would change my entire career trajectory. I was paired with Dr. Dorsa Ghoreishi, a mathematician at Saint Louis University who specializes in frame theory and phase retrieval. I then began learning about the field of frame theory, which sparked my passion for mathematics and would ultimately inspire me to want to become a professional mathematician and earn a Ph.D. in the subject.

While working with Dr. Ghoreishi, she introduced me to Dr. Sara Botelho-Andrade, a mathematician at my home university, the University of Denver, who also specializes in phase retrieval. I developed a close working relationship with Dr. Ghoreishi and Dr. Andrade to deepen my knowledge of frame theory and the underpinnings of functional analysis. These two women truly changed my life. Not only did they provide me with the space to learn mathematics, but they also taught me how to be a confident and resilient female mathematician by giving me a support system that I did not have before. With their guidance, I earned honors distinction in my major by completing a thesis investigating algorithms for phase retrieval using a frame-theoretic approach. I only hope that in my academic future, I have the chance to inspire young women just as my mentors did for me.

Part II: An Introduction to Finite Frames

In addition to having amazing mentors, frame theory and its applications inspired me to pursue a graduate degree in mathematics, so I would like to offer a brief introduction to finite frame theory. Hilbert space frame theory was first formally introduced by Duffin and Schaeffer in 1952 when studying problems in Fourier analysis. However, frame theory was popularized by Daubechies, Grossman, and Meyer in the 1980s who used frames for data processing applications [1]. Since then, frames have been used in a variety of applied settings, such as compressed sensing, signal processing, sampling theory, biomedical engineering, and quantum information theory, among many others.

Frame theory partly belongs to fields such as applied harmonic analysis and functional analysis and even utilizes tools from algebraic geometry and number theory. One of the reasons why I was drawn to frame theory is that only an elementary understanding of linear algebra is needed to understand the motivation behind the use of frames. A motivating idea in frame theory is to expand the notion of an orthonormal basis to make the recovery of vectors possible in the presence of erasure, and it added a new approach to the reconstruction of vectors both in theory and application. Frames, like orthonormal bases, provide a stable and continuous way to represent a signal (i.e., a vector), but allows for redundancy. More formally, we have the following definition:

Definition A collection of vectors $\Phi = \{\varphi_i\}_{i=1}^M$ in $\mathbb{R}^N$ (for $M \geq N$) is a **frame** if there exist constants $0 < A \leq B < \infty$ such that for every $x \in \mathbb{R}^N$,

$$A\|x\|^2 \leq \sum_{i=1}^M |\langle x, \varphi_i \rangle|^2 \leq B\|x\|^2,$$

where A and B are the lower and upper frame bounds, respectively. The frame is called a **Parseval frame** if $A = B = 1$ and an **equal norm frame** is $\|\varphi_i\| = \|\varphi_j\|$ for all $i, j \in \{1, \dots, M\}$.

A classical example of a frame in finite-dimensional space is called a Mercedes-Benz frame in $\mathbb{R}^N$ which belongs to the family of equiangular frames. This family of vectors is neither linearly independent nor orthonormal but maintains characteristics of orthonormal bases.

Example The Mercedes-Benz frame for $\mathbb{R}^2$ is the equal-norm Parseval frame given by

$$\left\{ \sqrt{\frac{2}{3}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \sqrt{\frac{2}{3}} \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}, \sqrt{\frac{2}{3}} \begin{bmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix} \right\}$$

and can be visualized in the following way when the frame vectors are rotated by 90 degrees and drawn in a circle with radius $\frac{\sqrt{3}}{2}$ as shown in Figure 1, see [2] for details.

As with orthonormal bases, frames provide a stable and linear method of recovering vectors in a Hilbert space. However, the advantage of using frames is that they are resilient to information loss and noise due to the redundancy in measurements. Before we can describe the reconstruction process using frames, we need the following definition that introduces a fundamental operator in the literature:

Definition The **frame operator** associated with the frame $\Phi = \{\varphi_i\}_{i=1}^M$ is defined by the map $S : \mathbb{R}^N \rightarrow \mathbb{R}^N$ with

$$Sx = \sum_{i=1}^M \langle x, \varphi_i \rangle \varphi_i \text{ for all } x \in \mathbb{R}^N.$$

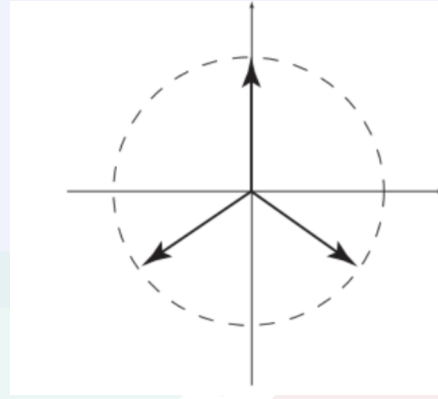


Figure 1: Mercedes-Benz frame

A crucial property of the frame operator is its invertibility because this allows us to be able to reconstruct vectors in $\mathbb{R}^N$. More precisely, any vector $x \in \mathbb{R}^N$ can be written as:

$$x = SS^{-1}x = \sum_{i=1}^M \langle S^{-1}x, \varphi_i \rangle \varphi_i = \sum_{i=1}^M \langle x, S^{-1}\varphi_i \rangle \varphi_i. \quad (1)$$

A frame $\{S^{-1}\varphi_i\}_{i=1}^M$ that satisfies (1) is called the *canonical dual frame* to $\{\varphi_i\}_{i=1}^M$. The notion of reconstruction is common in applications, for instance in recovering images and audio signals, but is also seen in related mathematical problems in phase retrieval and dynamical sampling theory. These are two topics that I am excited to explore more in-depth in the upcoming years of my Ph.D. program and will hopefully become the basis of my graduate research.

Although this is just a glimpse of the basics of finite frame theory, this mathematical subject is personally exciting to me because it combines areas of pure math, such as functional analysis, but also has practical applications in topics such as quantum theory, machine learning, and optimal transport theory. As for the future, my professional goal after obtaining my Ph.D. is to pursue a career in academia, specifically focusing on collaborative research with other disciplines – such as computer science and physics– utilizing frame theory. This trajectory would not have been possible without the mentorship of Dr. Ghoreishi and Dr. Andrade, and I hope to pay back that mentorship in the future.

References

- [1] P. Casazza, G. Kutyniok, and F. Philipp. Introduction to finite frame theory. *Finite frames: theory and applications*, pages 1–53, 2013.
- [2] P. Casazza and R. Lynch. A brief introduction to hilbert space frame theory and its applications. *Finite Frame Theory: A Complete Introduction to Overcompleteness*, 93(1):2, 2016.

The Critical Impact of Sexism in STEM

By Melissa Ochoa

Assistant Professor, Department of Women's and Gender Studies, Saint Louis University

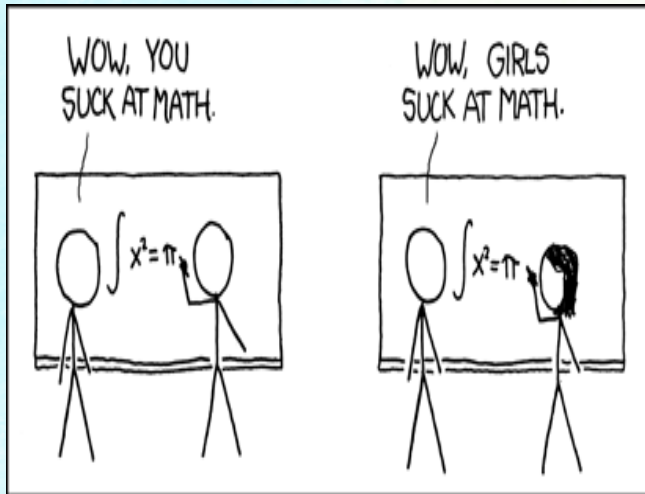


Figure 1: Comic from xkcd.com

Comic demonstrates “confirmation bias” in stereotype-consistent events (“women are bad at math”), assigning the stereotype to the entire group rather than the individual.

“Labs are filled with 30 guys and 5 girls. In such a male-dominant environment, I sometimes get the feeling that even though my point may be valid, I would not want to speak it for fear of being wrong and risking the male ego to think [because of my gender]: ‘that is why she is saying the wrong thing.’” (Huong, Asian woman)

I am not in STEM. I am a sociologist researching the everyday forms of sexism experienced by women in different social contexts—including in STEM fields. In my study, participants from two top-ranked engineering universities were asked to describe any interaction they perceived as an example of sexism. It yielded almost 1400 entries. I was not surprised when accounts flooded in from female² STEM students, but I underestimated the extent and the severity. Their experiences with sexism are consequential to the STEM fields and ultimately, society.

While most of the sexism reported was perpetrated by male classmates and male friends, a few accounts

demonstrated STEM professors and TAs also engaging in sexist discrimination. For example, Gabby [Latina] received a lower grade than her group members (all men) on their lab project and went to discuss this discrepancy with the male professor. His response? Well, she didn’t get as high of a grade because she was a girl.

Unfortunately, male peers weren’t any better, and perhaps, were even worse. They believed (and outrightly stated to women) that women were biologically less capable of being successful in STEM fields as “scientific fact.” They probably don’t realize that the original computers, computer scientists, and coders were, in fact, women.

“Five of us were [in the math help room] to study for my calculus class. I was sitting at a table with three guys and there was a girl at the other table. One of the [White] guys asked another [White] guy if he knew how to do a problem. When that guy didn’t know how to do it, he asked the [Chinese] guy if he could do it. [He] did not know how to do it either. The three of them discussed it for a little bit and gave up... The first guy never asked me or the other girl if we knew how to do it... I am usually good at math and I may have known how to do the problem, but they wouldn’t even ask. They would rather just not know than see if I or the other girl knew how. The three guys did not seem to know each other before this encounter, so it wasn’t just because they didn’t know us that they didn’t ask. The only difference between [us] and those three guys was our gender.” (White woman)

Until I reviewed the data, I didn’t realize how central group work was to STEM courses. Particularly, students were often assigned the same group for the semester to work on one or multiple projects. As male-dominated majors, women often reported being the only women in their groups and one of a handful in their courses and labs.

“In my precalculus class, we have a group project, and the professor asked us to pick groups of 3 to work on it throughout the semester... My partners said that males tended to be smarter at math and the females were better at time management. I found myself in a 2 male 1 girl team

ratio” (Latina)

No one likes to be excluded, rejected, or ostracized, but this was a common experience for female STEM students in their courses requiring group work. Pavan [Indian man] witnessed this firsthand in his computer science course during the process of selecting groups for their final project stating:

“...One [White] girl was looking for a group to join. When approached to ask whether or not she could join a group [consisting of three Asian male students], they said ‘no’ to her; but shortly after, another [White] male asked if he could join, and they let him in.” (Pavan, Indian man)

Carlos [Latino] revealed that guys in his engineering class explicitly said they didn’t want to work with girls. Male group members and classmates not only excluded their female classmates from directly working on their hands-on projects, but they also often assigned them the tasks of “taking notes,” “checking over their completed work,” or “bringing snacks.”

“...the [White] males in my group decided to do all the work while leaving the females out to do the notetaking. . . I started to volunteer in doing things that the [biology] lab entailed, but the males in my group had other plans, they stated ‘Why don’t you just record the data we tell you so that we can finish fast? We got it.’” (Keisha, Black woman)

Some of these women described such intense invisibility from their group members that they would just leave the group meetings or the classroom altogether, especially women of color who already felt marginalized at PWIs. Reyna [Latina] was “pushed away from participating [in building the robot]” by her group consisting of White men. She “left early because [she] felt excluded for the sole fact of being a woman.”

How successful can you be in your course when you are actively excluded from participating in projects consisting of lab work, constructing robots, or solving math worksheets? How easy is it to pass a course when you cannot participate in the learning process needed to advance in the major?

“I told my team I knew how to [turn on one of the lights on the microcontroller]. They completely ignored me and decided to figure it out themselves. . . they even talked to the other males sitting near us. . . They did not ask me once. . . I then just sat there silently and left the class.” (Colleen, White woman)

Female students are not “choosing” to leave STEM majors; they are being pushed out. Moreover, girls are being discouraged from pursuing STEM at all. By 8 years

old, girls think boys are better in STEM even when they score higher. The girls who push through and pursue a STEM major are often pushed out at the university level as well as the industry level.

These experiences from female STEM students are almost identical to what women faculty in STEM and women in the STEM industry report. For example, women at all three levels have encountered male colleagues commenting on their physical appearance, specifically on their attractiveness. Though completely inappropriate, a male student in an engineering study group told his female classmate she would make a “pretty stripper.” It seems to be mystifying to some men that women in STEM could be both “smart and attractive” as if those characteristics are mutually exclusive.

A few years ago, I was asked to present some of my data to a group of scholars and industry leaders seeking a large research grant to study sexism in STEM fields at the industry level. I made the point that we needed to start interventions much earlier, but at minimum, intervene at the university level for undergraduate students. During the Zoom presentation, I was asked to pause because female faculty in STEM and industry were not only reliving this discrimination but still experiencing it at their senior levels. They found it difficult to learn that female students in STEM were still experiencing sexism and had little to no resources to help them through it. These women were not in positions of power to take on this charge, but this burden should not be theirs alone to bear.

So how does this relate to Saint Louis University? I didn’t collect this data at SLU, but research on gender inequality and anecdotal experiences from women at SLU suggest the findings would be similar. In Fall 2022, I presented this research in the Department of Women’s and Gender Studies, and only two men showed up: a philosophy professor and a male student (major unknown). The women attendees shared that they had emailed the flyer, posted the flyer, and discussed it within their STEM departments, but none of their male colleagues showed up. Maybe it was the date and time? Well, I presented on a Sexism in STEM panel in Spring 2023 at a different time slot, and again, most attendees were women. Maybe this is just a coincidence? I won’t pretend to know how to do the probability here - I am a qualitative scientist. However, a couple of female engineering students told me that their male classmates recommended my “Sexism in STEM” presentation flyers to them as if it could only benefit women, not men, causing one to wonder if a similar sentiment is being shared throughout the fields.

Does gender inequity in STEM exist at SLU? It sure does. It exists in academia and the industry. Not only is it unjust, but it is a disservice to our communities and society

at large. We have lost, and are losing, so many important contributions to science and society because sexism in STEM persists. While SLU is not responsible for the existence of sexism in STEM, they should be accountable for the sexism women in STEM experience on campus by being proactive in eliminating it, committing to gender equity, and creating a space where everyone can learn productively regardless of gender identity. After all, *cura personalis*.

ⁱ Note: “Male” and “female” are not gender identities, but participants (and the general population) use them interchangeably with “men” and “women,” respectively.

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Hortensia Soto
(b. 1960s)

Soto was born in Mexico and grew up in western Nebraska. She is professor at Colorado State University, having previously taught at the University of Southern Colorado, and at the University of Northern Colorado. Her research and extensive publications center on the teaching and learning of undergraduate mathematics, with an eye towards equity in the classroom. Soto co-founded and directed the program *Las Chicas de Matemáticas*, an outreach program for mathematically talented high school girls, many of whom are headed towards being first-generation college students. Soto received an MAA Meritorious Service Award (2012) and an MAA Haimo Award (2018).

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

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Jacqueline Akinpelu
(b. 1953)

Akinpelu is an intelligence systems engineer at the Johns Hopkins University Applied Physics Laboratory (JHUAPL), where she has built an outreach program with Morgan State University. Before that she spent a quarter of a century with AT&T, where she led teams of mathematicians developing strategies for maintaining the long-distance network's stability under all conditions. Akinpelu is known for her research and leadership accomplishments in industrial applied mathematics. She also had a great influence on improving the workplace environment at AT&T and shaped their minority recruitment program. She received the Women of Color Technology Award for Career Achievement in Government (2009).

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Toric Varieties and Their Combinatorial Aspects

By Özlem Uğurlu

Assistant Professor, Department of Mathematics and Statistics, Saint Louis University

Throughout my master's research, I got really into toric varieties - these interesting algebraic objects defined by some combinatorial elements. The more I looked into them, the more I realized how cool their math structure is and how they're connected to combinatorics. Digging into these objects not only helped me grasp toric varieties better but also introduced me to the interesting side of combinatorics. It was quite a journey filled with curiosity and academic satisfaction.

In this article, I want to break down the basics of affine toric varieties for you. The main idea is to show how the combinatorial information of toric varieties makes it easier to work with and compute them. If you want more details, I encourage you to look at the toric variety books authored by David Cox [3] and William Fulton [4], the main resources I used to prepare this.

Introduction

Algebraically speaking, toric varieties often arise as varieties parameterized by monomials, and their structure is well-studied through objects from geometric combinatorics.

Before we start creating a toric variety combinatorially, let's set up a few notations and throw in some essential terms. If you have taken abstract algebra, you may recall that every finitely generated free abelian group N is isomorphic to $\mathbb{Z}^n$ for some positive integer n , and the isomorphism is equivalent to choosing a basis for N . It turns out that its dual - the group of homomorphisms from N to $\mathbb{Z}$, M , is also isomorphic to $\mathbb{Z}^n$. If that's not clear, just imagine N and M as xy -plane, where every point is an integer. When expressed as integer vectors, elements of N will be row vectors and those of M column vectors. You can take these concepts a step further by extending them to real vector spaces, which will be denoted by $N_{\mathbb{R}}$ and $M_{\mathbb{R}}$, respectively. These spaces can be thought of and treated much like $\mathbb{R}^n$.

Let $S = \{u_1, u_2, \dots, u_n\}$ be a finite set of vectors from $\mathbb{R}^n$. Then a cone that is closed under linear combinations with positive coefficients is called **convex polyhedral cone**. More precisely, a convex polyhedral cone is $\sigma = \{u \in \mathbb{R}^n \mid u = \sum_{i=1}^r \lambda_i u_i, \lambda_i \in \mathbb{R}_{\geq 0}\}$. Here, the vectors u_i 's are called **generators** of σ . In $\mathbb{R}^2$, we have standard basis $\langle e_1 = (1, 0), e_2 = (0, 1) \rangle$. By using this basis we can generate the following cones in $\mathbb{R}^2$, see Figure 1, Figure 2, and Figure 3.

$$0 \bullet \sigma_1$$

Figure 1: Example of a cone: $\sigma_1 = \{0\}$

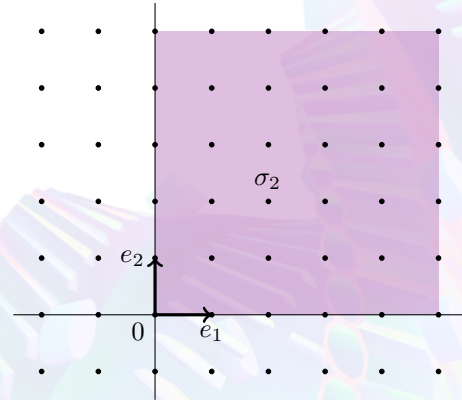


Figure 2: Example of a cone: $\sigma_2 = \langle e_1, e_2 \rangle$

A cone σ is a **rational cone** if all generators $u_i \in S$ of σ belong to N , meaning integer coordinates. Additionally, σ is a **strongly convex rational polyhedral cone** if it does not contain any straight line going through the origin. All the examples given above are strongly convex rational polyhedral cones and these objects are the building blocks for the construction of toric varieties. To keep things simple, we'll refer to a strongly convex rational polyhedral cone as just a "cone" from now on.

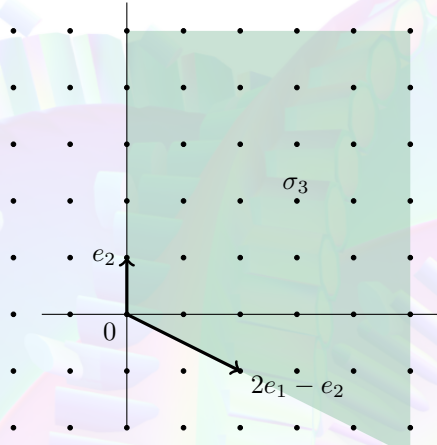


Figure 3: Example of a cone: $\sigma_3 = \langle e_2, 2e_1 - e_2 \rangle$

Next, we are going to define the dual of a cone, which can be considered as a mirror reflection of a cone. Loosely speaking, the dual cone is formed by the vectors perpendicular to each generator of the original cone. To be more precise, let $(\mathbb{R}^n)^*$ be the dual space of $\mathbb{R}^n$ and $\langle \cdot, \cdot \rangle$ the dual pairing. Then for a given cone σ its **dual cone** defined as to be a set

$$\check{\sigma} = \{v \in (\mathbb{R}^n)^* \mid \langle u, v \rangle \geq 0, \text{ for all } u \text{ in } \sigma\}.$$

Here, the symbol $*$ is used to differentiate between the original and its dual. See Figure 4, Figure 5, and Figure 6.

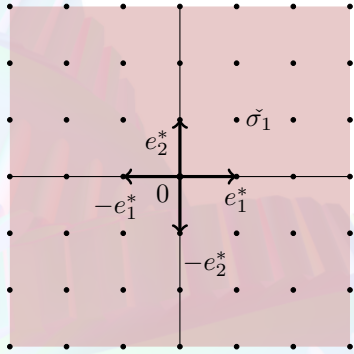


Figure 4: Dual of $\sigma_1 = \{0\}$ in $\mathbb{R}^2$

Construction of Affine Toric Variety

Let us start the construction process by building up our first concept, “affine semigroup”. A **semigroup** is a non-empty set S with an associative operation $+$: $S \times S \rightarrow S$. To be an **affine semigroup**, we further require that:

- (a) the binary operation on S is commutative,
- (b) S is finitely generated,
- (c) S can be embedded into a lattice.

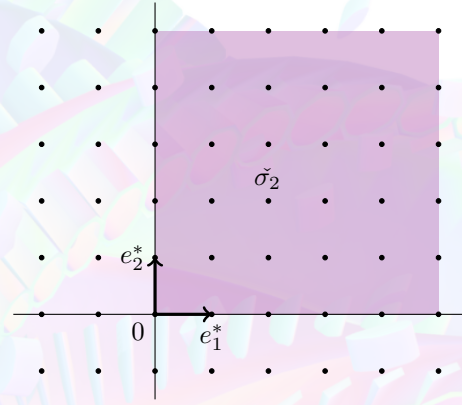


Figure 5: Dual of σ_2 in $\mathbb{R}^2$

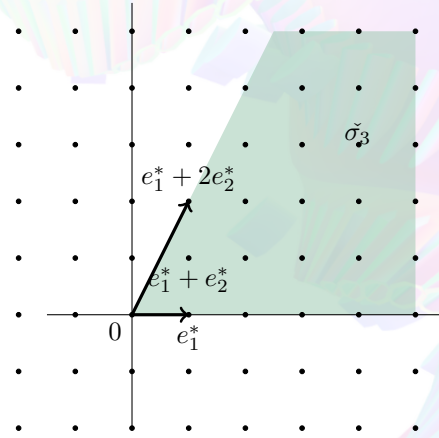


Figure 6: Dual of σ_3 in $\mathbb{R}^2$

$\mathbb{N}^n$ is an example of an affine semigroup. In a more general setting, for a given lattice L and finite set $\mathcal{F}$, the associated affine semigroup is in the form of $\mathbb{N}\mathcal{F} \subseteq M$. In fact, up to isomorphism, all affine semigroups are of this form. For a cone σ , we can define the corresponding affine semigroup S_σ as follows:

$$S_\sigma = \check{\sigma} \cap \mathbb{Z}^n = \{v \in (\mathbb{Z}^n)^* \mid \text{for all } u \in \sigma, \langle u, v \rangle \geq 0\}.$$

Simply speaking, this is nothing but a set consisting of integer points within the cone. The semigroup S_σ is, in fact, finitely generated (by *Gordon's Lemma*), which is a key condition in the theory of toric varieties.

Example In $\mathbb{R}^2$, consider the cone $\sigma_1 = \{0\}$ and its dual $\check{\sigma}_1$ generated by the vectors $e_1^*, -e_1^*, e_2^*, -e_2^*$ from the Figure 4. In this case, the semigroup $S_\sigma = \check{\sigma} \cap \mathbb{Z}^n$ is generated by the same vectors and we get $S_\sigma = \langle e_1^*, -e_1^*, e_2^*, -e_2^* \rangle$.

Example In $\mathbb{R}^2$, consider the cone $\sigma_2 = \langle e_1, e_2 \rangle$ and its dual $\check{\sigma}_2$ generated by the vectors e_1^*, e_2^* from the Figure 5. In this case, the semigroup $S_\sigma = \check{\sigma} \cap \mathbb{Z}^n$ is generated by the same vectors and we get $S_\sigma = \langle e_1^*, e_2^* \rangle$.

Example In $\mathbb{R}^2$, consider the cone $\sigma_3 = \langle e_2, 2e_1 - e_2 \rangle$ and its dual $\check{\sigma}_3$ generated by the vectors $e_1^*, e_1^* + 2e_2^*$ from the

Figure 6. In this example, $S_\sigma = \tilde{\sigma}_3 \cap M$, is not generated by the vectors e_1^* and $e_1^* + 2e_2^*$ alone. To obtain a set of generators, we also need to add $e_1^* + e_2^*$. Then, $S_{\sigma_3} = \langle e_1^*, e_1^* + e_2^*, e_1^* + 2e_2^* \rangle$. See [1] for more details.

As a next step, we try to understand how to construct the associated polynomials to the given semigroup, i.e. semigroup algebra. For a given cone σ , the associated semigroup algebra

$$\mathbb{C}[S_\sigma] = \mathbb{C}[\tilde{\sigma} \cap M]$$

consists of linear combinations of monomials χ^m , with multiplication given by $\chi^m \cdot \chi^{m'} = \chi^{m+m'}$.

Example Consider an n -dimensional cone σ generated by a basis $e_1, \dots, e_n$ of $\mathbb{Z}^n$ in which all coordinates are nonnegative. Then all coordinates of the basis $e_1^*, e_2^*, \dots, e_n^*$ of the dual cone $\tilde{\sigma}$ are nonnegative as well. This allows us to identify $\mathbb{C}[S_\sigma]$ with the usual polynomial ring $\mathbb{C}[X_1, \dots, X_n]$ by setting $X_i = \chi^{e_i^*}$. In particular, for the cone σ_2 from the Figure 2, the corresponding semigroup algebra is $\mathbb{C}[X_1, X_2]$.

Example Consider $\sigma = \{0\}$ in $\mathbb{R}^n$ with a dual the dual cone $\tilde{\sigma} = (\mathbb{R}^n)^*$, so that $S_\sigma = (\mathbb{Z}^n)^*$. Then the semigroup algebra $\mathbb{C}[S_\sigma]$ can be identified with the ring of Laurent polynomials $\mathbb{C}[X_1^{\pm 1}, \dots, X_n^{\pm 1}]$ by setting $X_i = \chi^{e_i^*}$ and $X_i^{-1} = \chi^{-e_i^*}$.

Thus, for a given cone σ , each element of the associated semigroup algebra $\mathbb{C}[S_\sigma]$ is of the form $f = \sum c_i \chi^{v_i}$ for $c_i \in \mathbb{C}$ and $v_i \in S_\sigma$. We are now able to define the affine toric variety associated with a cone σ .

In general, one can write

$$\mathbb{C}[S_\sigma] \simeq \mathbb{C}[x_1, \dots, x_N] / \langle f_1, \dots, f_s \rangle.$$

Then the affine variety

$$X_\sigma = \mathbf{V}(f_1, \dots, f_s) \subset \mathbb{C}^N$$

is the **affine toric variety** determined by the strongly convex rational polyhedral cone σ . Here, $\mathbf{V}(f_1, \dots, f_s)$ refers to the set of common zeros of a collection of polynomials $\{f_1, \dots, f_s\}$ and is called a **variety**. The term “affine” can be considered as a space in which everything works ‘nicely’.

Example Consider the cone $\sigma_1 = \{0\}$ from the Figure 1 and the corresponding semigroup

$$S_{\{0\}} = \langle \pm e_1^*, \pm e_2^* \rangle.$$

Setting

$$X_i = \chi^{e_i^*} \quad \text{and} \quad X_i^{-1} = \chi^{-e_i^*}$$

$$X_1 = x_1, X_2 = x_2, X_1^{-1} = x_3, \text{ and } X_2^{-1} = x_4$$

we get the following equations:

$$X_1 X_1^{-1} = 1 \text{ gives us } x_1 x_3 - 1 = 0$$

and

$$X_2 X_2^{-1} = 1 \text{ gives us } x_2 x_4 - 1 = 0.$$

Then the corresponding $\mathbb{C}$ -algebra is

$$\mathbb{C}[S_\sigma] \simeq \mathbb{C}[x_1, \dots, x_4] / \langle x_1 x_3 - 1, x_2 x_4 - 1 \rangle$$

Thus, $X_\sigma = \mathbf{V}(x_1 x_3 - 1, x_2 x_4 - 1)$. This affine toric variety corresponds to $(\mathbb{C}^*)^2$.

Example Consider the cone $\sigma_3 = \langle e_2, 2e_1 - e_2 \rangle$ from the Figure 3 and the corresponding semigroup $S_\sigma = \langle e_1^*, e_1^* + e_2^*, e_1^* + 2e_2^* \rangle$. Setting

$$X_1 = \chi^{e_1^*} \quad \text{and} \quad X_2 = \chi^{e_2^*}$$

$$X_1 = x_1, X_1 X_2 = x_2, \text{ and } X_1 X_2^2 = x_3$$

we get the following equation:

$$X_1 \cdot X_1 X_2^2 - (X_1 X_2)^2 = 0 \text{ gives us } x_1 x_3 - x_2^2 = 0.$$

Then the corresponding $\mathbb{C}$ -algebra is

$$\mathbb{C}[S_\sigma] \simeq \mathbb{C}[x_1, x_2, x_3] / \langle x_1 x_3 - x_2^2 \rangle.$$

Thus, $X_\sigma = \mathbf{V}(x_1 x_3 - x_2^2)$. This affine toric variety corresponds to the quadric cone in $\mathbb{C}^3$, see Figure 7.

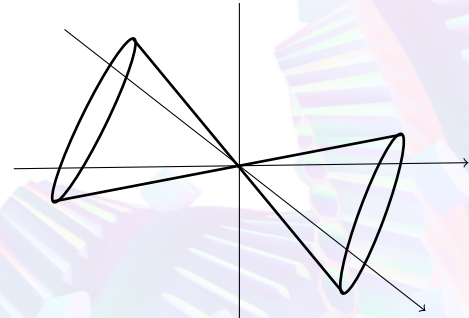


Figure 7: Real part of a quadric cone

Smoothness of Affine Toric Varieties

Studying singularities; in other words, investigating the behavior of points, of a given variety is a crucial aspect of algebraic geometry. It helps us to understand the structure of a variety better and also helps us to classify different types of varieties. Additionally, it has applications in other fields, such as algebraic coding theory, physics,

and engineering. For example, in mathematical physics, toric varieties have become relevant in the theory of mirror symmetry as they provide a way to understand the geometry of Calabi-Yau manifolds. This is important because the singularities of Calabi-Yau manifolds and their smoothings are important for understanding the mirror quintic. Furthermore, the connection between intersection numbers in toric geometry and charges in the gauge linear sigma models is described in [5]. Many other interesting applications of toric varieties can be found in [2].

We finish our article with the fact that a singularity of an affine toric variety can be determined by the associated cone.

Definition A strongly convex rational polyhedral cone in N is called **smooth** or **regular** if it is generated by part of a basis for N .

A strongly convex rational polyhedral cone in N is called **simplicial** if its generators are linearly independent over $\mathbb{R}$.

The cone pictured in Figure 2 is smooth while the cone in Figure 3 is simplicial.

Theorem 1. *An affine toric variety X_σ is smooth if and only if σ is smooth. Furthermore, all smooth affine toric varieties are of this form.*

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Jo Boaler
(b. 1964)

Boaler was educated in her native England, and taught at the secondary level before completing her graduate work there. Most of her subsequent career has been spent at Stanford University, where she is Nomellini-Olivier Professor of Mathematics Education. Her work focuses on approaches to learning mathematics that lead to student success, particularly active learning strategies and promotion of a

growth mindset, and her research has received media attention. Boaler is co-founder and faculty director of youcubed, a math education center that offers resources for teachers and students. She has received an NSF Presidential Early Career Award (2000), the Marie Curie Foundation Chair of Excellence (2007), and the NCSM Equity Award (2014).

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Lai-Sang Young
(b. 1952)

Young is from Hong Kong, and was educated in the US. She is the Henry & Lucy Moses Professor of Science at NYU's Courant Institute of Mathematical Sciences. Her interests are in ergodic properties of dynamical systems, computational modeling, and theoretical neuroscience. She has supervised twelve PhDs, three of them by women. Young's awards include a Sloan Fellowship (1985–86), the AMS's Ruth Lyttle Satter Prize (1993), and a

Guggenheim Fellowship (1997–98). She has given two invited talks at the International Congress of Mathematicians (1994, 2018), and been elected a Fellow of the AAAS (2004). Young is now a member of the National Academy of Sciences (2020).

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The Black-Scholes Model: Mathematics Explains the World of Finance

By Samuel Liu

Undergraduate Student, Department of Mathematics and Statistics, Saint Louis University

Introduction

The history of options can be traced back to the 1800s in Europe. Trading options has been a way for two parties to facilitate a potential transaction involving a set time, price, and asset. This concept can be simply understood using an analogy of an asset becoming a car.

Imagine it is January 1st, 2025, and you are given the opportunity to purchase a car on July 1st, 2025 at a strike price K of \$60,000. This car is a rare, vintage Mustang, and so, you believe the value exceeds the \$60,000 you would pay on July 1st. Fast forward to July 1st; at the end of the day, you must decide whether or not you want to buy the Mustang. However, there is a catch. In order to obtain the opportunity to buy the Mustang, you must pay a monthly premium to hold onto the option of purchasing the car or asset, at the discounted price on July 1st. If you decide to proceed, you must pay a \$1,000 premium each month for the six-month time to expire $\tilde{T}$. This means that securing the opportunity to exercise the option would cost you \$6,000 in premiums. This ability to buy the asset at the strike price is called “calling an option”.

After you call the option, you will likely be in one of two scenarios. The first occurs when you see that the market price of the Mustang has risen to \$100,000. In this case, you are “in the money”, meaning it would be profitable to exercise the option to buy the Mustang at the strike price. So, you decide to exercise your right as the holder of the option to purchase the Mustang at the \$60,000 price. Immediately after purchasing it, you flip the car, making a profit of \$34,000 (\$100,000 – \$60,000 – \$6,000). Now let’s consider the second scenario. Rewind to the beginning of the day (still on July 1st). In this case, the market price of the Mustang has dropped to \$40,000, putting your option “out of the money”. This situation indicates that exercising the option would result in a loss. It would be unwise to purchase this car, and consequently, you let your option expire, resulting in a loss equal to the total premium paid, which is \$6,000.

The idea of an option was used for almost three hundred years before the Chicago Board Options Exchange standardized a market to trade options in 1973. Prior to the creation of the market, traders relied on primal intuitions of the market to price options, leading to a variety of techniques that brought disorder to how options should be priced. As such, three men, Fischer Black, Myron Scholes, and Robert Merton, revolutionized the finance field, defining the framework for option pricing in what they coined the Black-Scholes Model:

$$c(S_0, 0) = S_0 N(d_1) - e^{-r\tilde{T}} K N(d_2), \quad (1)$$

where S_0 is the current market price of the underlying asset, K is the predetermined price at which the option is exercised, $\tilde{T}$ is the time to expiration, σ is the volatility of the stock price and r is the risk-free interest rate.

Here, the buyer of a European call option has the right to purchase the underlying asset S at a given price K , the strike price of the option, at a given date $\tilde{T}$. The problem in exercising that right to purchase lies in the randomness found in the market’s movements, which can be attributed to the constant variables σ , volatility, and r , the risk-free interest rate. This phenomenon is calculated through the cumulative distribution function $N(\cdot)$ for d_1 and d_2 under a standard normal distribution. Put simply, $N(d_1)$ and $N(d_2)$ act as a probability weight on S and K , respectively. The payoff that stems from this probability is given in the equation below:

$$c(\tilde{T}) = \max(S(\tilde{T}) - K, 0).$$

In the case that the stock is exercised at price K , then $S(\tilde{T})$ would be greater than or equal to K , meaning the payoff would be considered *as in the money* or profitable. Otherwise, when $S(\tilde{T})$ is less than K , the payoff would be put *out of the money* or unprofitable. The model and formula presented are expanded upon through a comprehensive analysis in [4, Chapter 3] and [3, Chapter 10]. These texts provide a thorough understanding of the elegance within the mathematics of pricing options.

The following graph illustrates the profitability of an option at various market prices. Observe that if the market price is below or at the strike price, the profit is \$0 as the option is either ‘out of the money’ or at break-even.

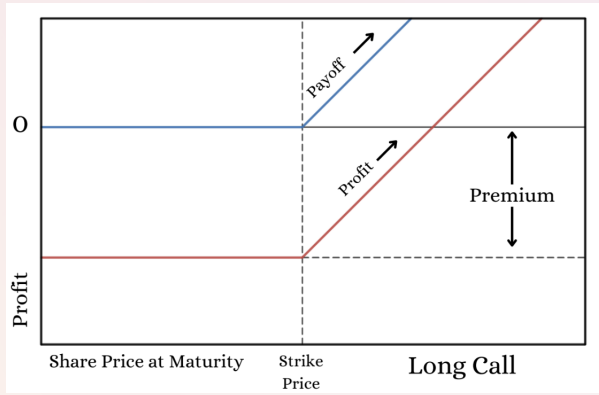


Figure 1: Profits on a European Call Option [1]

Mathematical Analysis of the Black-Scholes Equation

A European call option is a solution to a partial differential equation called the Black-Scholes equation (BSE). The Black-Scholes equation is a second-order partial differential equation that evaluates the price of a financial derivative, represented as:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0, \quad (\text{BSE})$$

Each term in this equation encapsulates a critical aspect of the financial market's behavior:

1. $\frac{\partial V}{\partial t}$ is “time decay”. So, as time approaches the expiration date, the value of the option will decrease.
2. $\frac{\sigma^2}{2} S^2 \frac{\partial^2 V}{\partial S^2}$ is the measure of how the option bends and curves in relation to changes in the underlying asset's price.
3. $rS \frac{\partial V}{\partial S}$ is the benefit or cost of holding the underlying asset throughout the lifetime of an option, adjusted for the risk-free rate.
4. $-rV$ is the opportunity cost of holding an option rather than investing in the risk-free stock over the same period of time.

Partial differential equations (PDEs) hold an infinite number of solutions. However, under boundary conditions, some PDEs will be confined to one solution. This is precisely the case of the Black-Scholes PDE; each set

of boundary conditions is a snapshot of the dynamic market. The BSE interconnects numerous market variables to calculate European option prices: the current price of the underlying asset S , time to expiration $\tilde{T}$, predetermined price at which the option is exercised K , risk-free interest rate r , and volatility of the stock price σ (unit = $1/\sqrt{\text{time}}$). The price of the option satisfies the boundary values $V(0, t) = 0$ for $0 \leq t \leq \tilde{T}$ and at time $t = \tilde{T}$, $V(S, \tilde{T}) = \max(S - K, 0)$ and $V(S, t) \sim S$ (i.e. $V(S, t)/S$ tends to a constant as $S \rightarrow +\infty$).

With numerous constants and conditional factors to consider, the BSE has become quite complicated to start to conclude generalities by itself, especially having variables that are bound by different scales. This complexity requires an approach that can reveal the underlying structural dynamics between relationships: non-dimensionalization. Non-dimensionalization is a technique that removes the physical attributes of variables by simplifying their parameters; it enables mathematicians to look past the limitations of specific units to observe patterns in the purest form. The power of non-dimensionalization paves the way for a transformation to the well-known Heat Equation. Such a transformation resembles uncovering the hidden intricacies of a complex mosaic; the solution is yielded from the Heat Equation:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0. \quad (\text{Heat-eq})$$

To simplify the notations, we will represent partial derivatives using subscripts: for instance, $\frac{\partial u}{\partial t} := u_t$ or $\frac{\partial^2 u}{\partial x^2} := u_{xx}$. The process is as follows:

Non-dimensionalization. To get a partial differential equation without a unit, we introduce the new variables $\tau = \frac{t}{T}$ and $U = \frac{S}{K}$, where T is the characteristic time scale parameter and K is the characteristic price scale parameter. The two constant T will be determined later in the process.

Let $E(U, \tau) = \frac{1}{K} V(S, t)$. Equation (BSE) is expressed with the new variables τ and U as:

$$\frac{1}{T} E_\tau + \frac{\sigma^2}{2} S^2 \frac{1}{K^2} E_{UU} + rS \frac{1}{K} E_U - rE = 0.$$

Using the fact that $\frac{S}{K} = U$ and by factorizing by $1/T$, we obtain that

$$E_\tau + \frac{\sigma^2 T}{2} U^2 E_{UU} + rT U E_U - rT E = 0. \quad (2)$$

Now we need to assign a value to T (with a unit of time) in order to simplify the number of parameters involved in the

equation. For that, we set one of the constant coefficients equal to 1. For instance, we choose $\frac{\sigma^2 T}{2} = 1$ which leads to

$$T = \frac{2}{\sigma^2} \quad \text{and} \quad rT = \frac{2r}{\sigma^2}.$$

We group the two parameters r and in T by replacing rT by a new parameter $k = rT$. Equation (2) becomes:

$$E_\tau + U^2 E_{UU} + kU E_U - kE = 0. \quad (3)$$

The condition $V(S, \tilde{T}) = \max(S - K, 0)$ changes with the substitution and becomes

$$\begin{aligned} E\left(U, \frac{\tilde{T}}{T}\right) &= \frac{1}{K} V(S, \tilde{T}) \\ &= \frac{1}{K} \max(S - K, 0) = \max\left(\frac{S}{K} - 1, 0\right) \\ &= \max(U - 1, 0). \end{aligned}$$

Simplification of the equation. The coefficients of the partial derivatives in (3) are not constant, which is not the case for the heat equation. Observe that the coefficient of E_U is kU^1 , where the power of U in the coefficient matches the order of differentiation of E with respect to U .

This phenomenon occurs in ordinary differential equations (ODEs), particularly with Cauchy-Euler's equations. The power in this method is demonstrated through a logarithmic substitution, $x = \log(U)$. Substituting $x = \log(U)$ simplifies the complexity of the equation, turning variable coefficients into constant ones. For that, set $x = \log(U)$ and $g(x, \tau) = E(U, \tau)$. Then, g satisfies the following partial differential equation:

$$g_\tau + g_{xx} + (k-1)g_x - kg = 0, \quad (4)$$

and the condition $E\left(U, \frac{\tilde{T}}{T}\right) = \max(U - 1, 0)$ turns into

$$\begin{aligned} g\left(x, \frac{\tilde{T}}{T}\right) &= E\left(U, \frac{\tilde{T}}{T}\right) \\ &= \max(U - 1, 0) \\ &= \max(e^x - 1, 0). \end{aligned}$$

Reduction to the Heat equation. The above equation is not yet in the form of the classical heat equation. The final step in the transformation to the heat equation starts with a strategic substitution:

$$g(x, \tau) = e^{-\alpha\tau - \beta x} \cdot H(x, \tau). \quad (5)$$

This substitution is pivotal. The strategic substitution, by multiplying g with the exponential factor $e^{-\alpha\tau - \beta x}$, is carefully chosen to counterbalance the terms τ and x , significantly simplifying the partial differential equation. With

this substitution, H emerges as the new subject of our equation:

$$\begin{aligned} g_\tau + g_{xx} + (k-1)g_x - kg &= \\ -\alpha \cdot e^{-\alpha\tau - \beta x} \cdot H + e^{-\alpha\tau - \beta x} \cdot H_\tau &+ \beta^2 \cdot e^{-\alpha\tau - \beta x} \cdot H - 2\beta \cdot e^{-\alpha\tau - \beta x} \cdot H_x + e^{-\alpha\tau - \beta x} \cdot H_{xx} \\ + (k-1)(-\beta \cdot e^{-\alpha\tau - \beta x} \cdot H + e^{-\alpha\tau - \beta x} \cdot H_x) &- k \cdot e^{-\alpha\tau - \beta x} \cdot H \\ = e^{-\alpha\tau - \beta x} [H_\tau + H(\beta^2 - \alpha - \beta(k-1) - k) &+ H_x(k-1-2\beta) + H_{xx}] \\ = 0. \end{aligned}$$

We now choose α and β to cancel the coefficients of H and H_x . Selecting $\beta = \frac{k-1}{2}$ and $\alpha = \beta^2 - (k-1)\beta - k = -\frac{(k+1)^2}{4}$, we are close to aligning our model with the heat equation:

$$H_\tau + H_{xx} = 0. \quad (6)$$

with the following boundary condition:

$$\begin{aligned} H\left(x, \frac{\tilde{T}}{T}\right) &= e^{\alpha \frac{\tilde{T}}{T} + \beta x} \cdot g\left(x, \frac{\tilde{T}}{T}\right) \\ &= e^{\alpha \frac{\tilde{T}}{T}} e^{\beta x} \cdot \max(e^x - 1, 0) \\ &= e^{\alpha \frac{\tilde{T}}{T}} \cdot \max(e^{x+\beta x} - e^{\beta x}, 0) \\ &= e^{\alpha \frac{\tilde{T}}{T}} \cdot \max(e^{(k+1)x/2} - e^{(k-1)x/2}, 0). \end{aligned}$$

To arrive at the heat equation, one needs to make one last substitution: let $\tilde{\tau} = \frac{\sigma^2 \tilde{T}}{2} - \tau$ and $\mathcal{H}(x, \tilde{\tau}) = H(x, \tau)$. Then, it is clear that equation (6) becomes

$$-\mathcal{H}_{\tilde{\tau}} + \mathcal{H}_{xx} = 0. \quad (7)$$

Now the (BSE) is reduced to the heat equation, it is possible to give an expression of $V(S, t)$ from the general solution of the heat equation. Theorem 1 gives an analytical expression of the heat equation: the discussion does not address the uniqueness or smoothness of the solution.

Theorem 1. *The solution of the heat equation (7) is given by*

$$\mathcal{H}(x, \tilde{\tau}) = \frac{1}{\sqrt{2\pi\tilde{\tau}}} \int_{-\infty}^{+\infty} \mathcal{H}(s, 0) e^{-(x-s)^2/(4\tilde{\tau})} ds.$$

In our case, the boundary condition for $\mathcal{H}$ is given by

$$\begin{aligned}\mathcal{H}(s, 0) &= H\left(s, \frac{\tilde{T}}{T}\right) \\ &= e^{\alpha \frac{\tilde{T}}{T}} \max(e^{(k+1)x/2} - e^{(k-1)x/2}, 0) \\ &= e^{-(k+1)^2 \sigma^2 / 8} \max(e^{(k+1)x/2} - e^{(k-1)x/2}, 0),\end{aligned}$$

since $\tilde{\tau} = 0$ when $\tau = \frac{\tilde{T}}{T}$. From the knowledge of the solution to the heat equation and the following relation

$$\begin{aligned}V(S, t) &= K \left(\frac{S}{K}\right)^{-(k-1)/2} \cdot e^{-(k+1)^2 \sigma^2 (\tilde{T}-t)/8} \\ &\quad \cdot \mathcal{H}\left(\log\left(\frac{S}{K}\right), \frac{\sigma^2}{2}(\tilde{T}-t)\right),\end{aligned}$$

we obtain the solution of the Black-Scholes equation $V(S, t)$.

Theorem 2. *The solution of the Black-Scholes equation is given by*

$$V(S, t) = SN(d_1) - Ke^{-r(\tilde{T}-t)}N(d_2),$$

$$d_1 = \frac{\log\left(\frac{S}{K}\right) + (r + \frac{1}{2}\sigma^2)(\tilde{T}-t)}{\sigma\sqrt{\tilde{T}-t}},$$

$$d_2 = \frac{\log\left(\frac{S}{K}\right) + (r - \frac{1}{2}\sigma^2)(\tilde{T}-t)}{\sigma\sqrt{\tilde{T}-t}},$$

where $N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^d e^{-\frac{1}{2}s^2} ds$ is the cumulative distribution function for the standard normal distribution.

Observe that for $S = S_0$ and $t = 0$, we find equation (1). We omitted the details to obtain the formula in Theorem 2. We refer to [2, Chapter 8] for more details.

Conclusion

The Nobel Prize in Economic Sciences in 1997 was awarded for the work behind the Black-Scholes model- a recognition that, along with an interest in quantitative mathematics, led me to start studying Black, Scholes, and Merton's work. I hope that the reader has been able to catch a glimpse of the elegance in their work. Moreover, I hope the reader has gained a sense of how financial derivatives operate both in the world itself and mathematically through the analysis. To conclude, the exploration of the Black-Scholes model not only marks an exceptional achievement, but also transcends the boundaries across mathematics, economics, and physics. Understanding the world is an interdisciplinary pursuit. Stay curious!

This work has been conducted under the expertise of Elodie Pozzi. I would like to extend my deepest gratitude to her; thank you Professor Pozzi for your unwavering support and invaluable advice throughout the past year.

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Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called “EvenQuads” to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.



Nkechi Agwu
(b. 1962)

Agwu is from Nigeria, and grew up there as well as in Sierra Leone. She worked as a government statistician and as a teacher at a vocational college before doing her graduate work in the US. She teaches at the City University of New York, and has also taught at the Borough of Manhattan Community College. Agwu's interests include gender studies, ethnomathematics, and the history of mathematics, especially the math of Ndebele dolls, African textiles, and the use of women's storytelling in STEM education. She has published two papers on Black mathematicians. She was awarded a Carnegie Africa Diaspora Fellowship (2014).

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.

EVENQUADS



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Students' Perspectives:

The William Lowell Putnam Mathematical Competition

Greg Marks, Ph.D., Associate Professor of Mathematics and Statistics, is the faculty supervisor of the Saint Louis University Putnam Competition team.

Marks: A Note from the Faculty Mentor

The William Lowell Putnam Mathematical Competition is the most prestigious intercollegiate mathematics contest in the United States and Canada. Now in its 86th year, the competition consists of an extremely challenging 12-problem, six-hour exam, conducted nationwide on the first Saturday of December each year. There is really no pressure, since participants remain anonymous unless they do well. The problem topics span the entire undergraduate curriculum and offer students a taste of what research in mathematics is like: you're confronted with a fascinating problem of a sort that you've never quite seen before, and you hope to use your ingenuity and accumulated mathematical knowledge to find a solution. Some problems are otherworldly: What is the largest circle you can fit inside a 4-dimensional cube? Some problems have practical applications: Can you simulate an arbitrary probability with flips of a fair coin? SLU has fielded a Putnam team every year since 2001, and students over the years, regardless of how they did in the competition, have spoken of how the competition helped prepare them for graduate school or for other academic contests, and that they simply enjoyed racking their brains working on memorable problems!

Nathaniel Alloway

One thing to keep in mind when solving Putnam problems is that seeing the solution is only half the battle - you have to write a sufficiently rigorous proof to back up whatever it is you are seeing. But it can be tricky to know how much detail is necessary to provide! In my case, when I was working on problem B1 from the 2023 exam, which involved counting the number of legal configurations obtained by performing a certain move on a board filled with coins, I was able to see that there was a one-one correspondence between legal configurations and paths on the board that only go up and right. Counting the number of paths and getting the right answer only takes a bit of elementary combinatorics, but demonstrating the correspondence proved considerably more difficult for me. I ended up spending about five pages of paper trying to clearly hammer out every step of the proof! By contrast, the solution linked

on the MAA website uses the same technique I used in my solution but is much more concise.

Bernadette King

The Putnam was a fun way to use my mathematical skills in a new way. I had never competed in a math competition, and it allowed me to use math in a creative way we typically don't inside the classroom. Aside from the mathematical component, it allowed me to test my patience and perseverance. It taught me how to allow myself grace as I am trying to understand difficult math problems.

Kieran Favazza

I took the Putnam for the third time this past year, and I always enjoy it. It takes up a Saturday, but it is nice to be reminded of all the math you don't know. I have spent time in years past trying to learn little tricks that can earn you solutions, but it ultimately comes down to creativity and confidence. I never solve that many problems, but I do my best to believe that I can do all of them. I try my best to be creative and see how the math of these clever problems can pervade my life and change my perceptions. The Putnam is a great way to learn corners of mathematics you have never seen, and it is a great exercise in trusting yourself and having grace. I look forward to taking it for the final time next year - I am excited to strengthen my mind and see how my mathematical development changes the way I view problems.

Mentoring Math Minds Program

The Department of Mathematics and Statistics at Saint Louis University is excited to announce the launch of a Research Mentoring Program tailored for both graduate and undergraduate students, with a strong Diversity, Equity, and Inclusion (DEI) component during the 2024-2025 academic year. This program is designed by Dr. Ghoreishi, Dr. Pozzi, and Dr. Uğurlu who are woman mathematicians from diverse cultures and share a dedication to mentoring and advising students in mathematics and fostering their professional development.

What makes this program special for participants are the opportunities to be involved in research projects, access to platforms to present their work, and receive financial support to attend workshops and conferences. Participants will also receive expert mentorship and explore a wide range of professional development opportunities, providing a comprehensive experience – all strategically chosen by the project leaders for a long-lasting integration of diversity initiatives.



Program Objectives

- Comprehensive training for all student participants.
- Access to valuable resources and professional development opportunities.
- Enhanced engagement of underrepresented groups in research activities.

Application Process

The application process will consist of two steps. All interested candidates are invited to complete a Google form for pre-application. Applicants will be asked to submit their resume, a letter of interest, and a DEI statement to finalize their application. We encourage a diverse range of applicants to contribute to the richness of our program.



Start your pre-application now!

Admission and Community Agreement

Upon admission, successful applicants will be requested to sign a Community Agreement, emphasizing the program's aspects and expectations. This agreement aims to foster a safe and supportive environment for all participants.

Join us in fostering a collaborative and inclusive research community at the Department of Mathematics and Statistics. Together, let's explore the exciting world of mathematics and statistics by creating a supportive, inspiring, and positive community for all!

For inquiries and application details, please contact one of the project leaders:

- Dorsa Ghoreishi - dorsa.ghoreishi@slu.edu
- Elodie Pozzi - elodie.pozzi@slu.edu
- Özlem Uğurlu - ozlem.ugurlu@slu.edu

MENTORING MATH MINDS PROGRAM 2024 – 2025



The Department of Mathematics and Statistics at Saint Louis University is thrilled to launch a dynamic year-long Research Mentoring Program tailored for both graduate and undergraduate students, with a strong Diversity, Equity, and Inclusion (DEI) component.

1

Two-Step Application

All interested candidates are invited to complete a Google form for Pre-Application. **(Apply by May 15th)**

Applicants then will be asked to submit their resume, a letter of interest, and a DEI statement to finalize their application.

We encourage a diverse range of applicants to enhance the richness of our program.

2

Program Benefits

Comprehensive training for all student participants.

Access to valuable resources and professional development opportunities.

Enhanced engagement of underrepresented groups in research activities.

**PRE-APPLICATION NOW
OPEN!**

Join us in fostering a collaborative and inclusive research community at the Department of Mathematics and Statistics. Together, let's explore the exciting world of Mathematics and Statistics by creating a supportive, inspiring, and positive community for all!

