

FOR ALL

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ABOUT US

Biannual inclusive mathematics magazine featuring articles and expositions on a wide range of topics related to mathematics and more.



Vol. 1, No 1 | March 2023

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Welcome to *For $\forall$ ll*

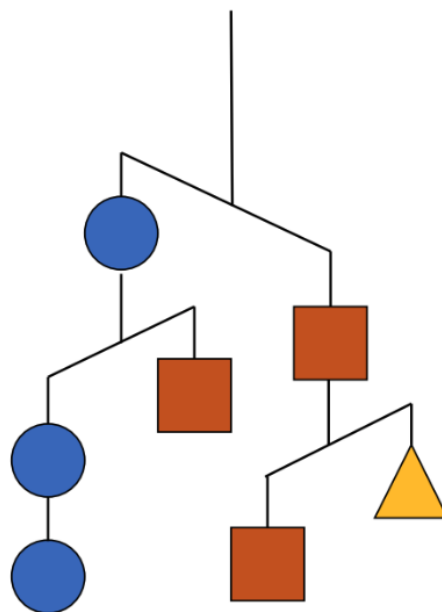
Welcome to *For $\forall$ ll*, the inclusive mathematical journal of Saint Louis University! We, faculty and students of the Department of Mathematics and Statistics at SLU, love St. Louis - it is our home, and we endeavor to make it a better place by creating inclusive communities that can share in the joy of learning and the city's teeming beauty. When we learn math together, we learn about each other, and when we learn about each other, we strengthen our minds and spirits in ways that transcend the mathematical puzzles at hand. In this way, we can gain a glimpse of understanding of each other - our diverse backgrounds and identities coalesce in our common desire to understand something so enormous and elusive as mathematics. To foster this learning, we have compiled a great selection of mathematical resources, including both informal and formal math articles, interviews with faculty, personal mathematical narratives, and fun puzzles. We thank our faculty and students for their contributions that make this magazine possible. We hope our offerings can bring together all lovers of knowledge - no matter their religious, sociopolitical, racial, ethnic, or mathematical background. This journal is an interdisciplinary one - while its content may be oriented towards mathematics, students, and researchers of all areas can find great elucidation in the many intersections between mathematics, our lives, sociopolitical issues, and research in every field. All who endeavor to persevere in learning deserve to share in its exuberance - help us to provide the framework in which we can all rejoice in our common love. Please enjoy our selection of content, and please submit your own! This is a journal of the people in our community,

and it is strengthened by any art, puzzles, or articles the community can provide.

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Which Shape is the Lightest? Which is the Heaviest?



Credit to Emily Twardy

Faculty Interview Series

Interviewing: Darrin Speegle

Interviewed by: Trevor Andershock and Kieran Favazza



Introduction

In the course of mathematical learning, we often find ourselves alone. Either we are stuck on a problem, racking our brains and cursing the math gods, or we feel like we're the only ones that don't get it - that everyone else can do what we can't. But this is simply not true - we are constantly surrounded by people who are just as confused as we are, no matter how it may seem, and our common love of the beauty of mathematics unites us in our venture into infinite knowledge. We can always ask another person for help on a problem, whether it's a friend, colleague, or instructor. Our friends and instructors are often the answer to our stubborn math problems - even if they do not give us the answer, their distinct minds move us to think about things in different ways, we eventually solve the problem, and we are fulfilled until the emergence of the next problem. And more often than not, the people in our lives are glad that we added to the beautiful confusion that is their

understanding of mathematics by simply asking them questions. For these reasons and for the fulfillment of our mathematical spirits, mathematics is not a pursuit meant to be undertaken alone. Dr. Darrin Speegle, professor in the Department of Mathematics and Statistics at Saint Louis University, is an illuminating reminder of these facts, and he shares great insights regarding this strange mathematical journey we all take.

Interview

Dr. Speegle, what made you become a mathematician?

I guess I've always liked math - ever since I was a kid (I think most mathematicians probably did not dislike it too much in childhood). My dad wanted me to go into electrical engineering when I got to college, but I don't think I liked the applied stuff as much, and the math classes appealed to me a lot, so I switched my major to math about halfway through my undergraduate education. I also kind of always wanted to be a professor, so that didn't really change.

In undergraduate school, did you have any idea of what kind of math you wanted to do?

I had no idea. I didn't even really know what math was; I was very naive at that time. I had taken calculus courses, differential equations, and linear algebra, so I was in for a surprise when I got to the more abstract proof-based classes. But I still liked it - it was just very different from what I was expecting.

What is it about being a mathematician now that you love?

I really like being able to work on problems in the long term. I like to find something that's interesting to me and just keep working on it. In other fields, what's important changes a lot more quickly than in math - the current state of knowledge is changing a lot more in other fields. I like the fact that these things that we prove are important tend to stay important for a long time.

I like working on problems with other people. I have a lot of fun working with colleagues that I've been

working with for years, and it's really fun to do that because you get to know these people, and it's so much better than just pounding away at these problems on your own.

Dr. Speegle stresses the importance of his collaboration with his colleagues. How could we rejoice at our struggles and discoveries without sharing them with our friends? Dr. Speegle says it's amazing to be around people that do research like him, even if it's not in the same area - everyone has something to learn from each other's problems, and it's fun and enlightening to just work on problems with friends.

What is your daily life like? Are you just thinking about math all the time?

These days, when I get home, I'm pretty much done with math for the day. When I was younger, I tended to just work on things and think about them all the time - I couldn't really turn it off. I would go to sleep thinking about a problem, I would dream about it, and I would wake up thinking about it. Nowadays, I'm definitely better at compartmentalizing things - especially because I have a family now, and I spend lots of time with them.

What are your hobbies outside of mathematics?

My biggest is probably ultimate frisbee. I still play around twice a week in the fall, and I'm on a team that travels to play tournaments around the country. I've even coached some high school teams, which was really fun. I love playing - it's probably the activity I enjoy most outside of math.

Dr. Speegle shares how he and his math have changed over time. He expresses his perception of his younger self as a little naive but always ready to do the math. He may have eaten, slept, and breathed mathematics, but it made him into the mathematician he is today. And now, he says that he has found the right balance in his life - he is there for his family and spends a great time with them, but he still has enough time to grapple with all the exciting problems that he loves - that has never changed.

What is your favorite math course that you ever took?

Probably the Analysis and Measure Theory course I took in graduate school. That was a lot of fun. It was super hard - on the first day, there were around 12-15 students, and I noticed that the class moved quickly and that I would need to put in some work for this class. It was a long class, so about halfway through, the professor gave us a short break. After the break, there were only like 7 people in the class. And among those 7, I was the only one to finish the course. So it was very hard, but it certainly prepared me to get my Ph.D. in analysis. So for me personally, it was an awesome class - I really learned a lot.

What is it about Analysis/Measure Theory that you like?

There was this one technique we used all the time - basically, if you're trying to prove something about this complicated thing, you kind of don't know where to start. But if you can prove something about this simple thing, you can use some basic tools to build up from that and prove the complicated thing. And that really appealed to me because I like proving simple things and learning the tools to go from simple to complicated was just really fun.

What have you found to be the best way to study mathematics?

The thing that really saved me in that hard graduate course was teamwork. I worked with another student in the class, and we got together every week to go over the theorems proved in class. We'd each bring in a theorem or two, go over them, understand them, and then try to prove them to each other without using our notes. With a few theorems a week, it was enough to understand just about all the important theorems in that class. And that really helped - it was great to have another person pushing me along. So working in teams is really an amazing way to study mathematics.

What were your biggest challenges as a student?

The biggest challenge as a student was the transition from taking classes to doing research. For me, that was really hard. I think I really undervalued personal relationships in that graduate school period - I wasn't proactive in developing relationships with advisors, so I struggled to really engage in the community of research. I was discouraged by my lack of ability to do research and the things that other people were doing, so I was ready to quit. I took some actuarial exams, and I was fully ready to become an actuary when one of my professors started talking about some problems in analysis. I was like, "Oh, I know I can at least do this," so I started working on them. I showed them to my professor, and he was really excited by my work, and he told me to keep working on them. Just that little bit of encouragement made me keep going - and now I have my Ph.D. in analysis.

Dr. Speegle speaks to an amazing part of mathematics and life in general - only the tiniest things are needed for our perseverance. All it takes is one good professor, one good classmate, one friend, or one chance encounter with something completely unrelated to make us realize the beauty and enormity of the things we love. Dr. Speegle shares how discouraged he was; how he felt like he wasn't cut out for mathematics. He felt like he wasn't part of the community that he so loved, but the help of one classmate and the encouragement of one professor revealed the unbreakable relationship between himself and this strange, daunting, yet unrelentingly powerful, all-encompassing world. That inexplicable, unfamiliar warmth of the call of the mathematical forces of nature -

revealed merely in the activity of knowing that someone believes in you - makes us try a little more than last time. And that seems to be all we need - just a little more.

Dr. Speegle's Research:

Like all great mathematicians, Dr. Speegle and his interests have evolved over time, so he has done research in a couple of different areas. His largest focus, however, has been on wavelets. Let us explore them.

Let's say that we need to represent some information (like songs on our phones, photos, or some other kind of data). We say that this information is contained in its own unique type of space (a vector space). So we wish to express information in the form of vectors. But we would like to use very little data to accomplish this, so it is in our interest to pick a couple of vectors that can generate the whole space of information that we need (in linear algebra, these couple vectors make up what is called a basis). But we want to pick this basis in such a way that everything will work out nicely; we don't want to have to do very much computation or use very large numbers to represent our information in terms of our basis vectors. A wavelet, then, is a way of constructing that basis. It's a little complicated to explain how the wavelets determine a basis, but let us first understand what a wavelet actually consists of.

Imagine we have some function or signal that we wish to encode. We encode it by approximating it with a wavelet function, so we start by constructing some functions. We are allowed to do two things to the wavelet to make it work: translate it (left and right) by integer values, or dilate it up and down by factors of 2. So we manipulate our wavelet, moving it left and right and dilating it up and down until it kind of looks like the function we're trying to represent. Now, we have a nice-looking function that encodes the information we need. We construct wavelets for a lot of different bits of information, and from here, there is a lot of analysis machinery to take our wavelet and make it into a basis that generates the "space" of all the information we need.

Dr. Speegle has done considerable research on wavelets and their properties - his favorite thing he's ever proved, he says, is the characterization of "wavelet sets" that can tessellate space. A tessellation is a collection of sets that can cover a space with no overlaps and no gaps - like the tiles on the floor in Ritter Hall or the honeycombs of a beehive. We can manipulate sets in many different ways to have them tessellate something, but Dr. Speegle, when he started his research over 20 years ago, was particularly interested in the types of sets that we can translate by integers and dilate by factors of 2 to tessellate space (do you see the connection between these and the wavelets yet?). As it turns out, these sets have many interesting properties, and we have quite a few tools to turn these sets into actual wavelet functions that can help us with our information problems. In

2020, Dr. Speegle concluded his research and came up with a comprehensive characterization of wavelet sets - he came up with a concrete method to determine which kind of sets can be wavelet sets. He is excited to have figured out such a great problem, and now he's continuing his research into wavelets, investigating their properties and ways to use them.

What are your strategies when figuring out a tough problem?

I think I have a couple that works really well for me. One was suggested to me by a professor: work on the easiest thing that you don't know. A lot of times, you have a goal in mind - something you want to understand. So you ask yourself, "What do I need to understand in order to understand this thing?" You keep going until you find something that you don't understand, and that's where you start. You understand that and then work your way back up to your end goal. A lot of times, it's helpful just to say, "Okay, what would this complicated thing be like in this very simple case?" Sometimes, it's not that interesting, but sometimes, it's quite helpful.

Another thing I like to do is to just come up with examples. If I have a theorem I don't understand or that I'm trying to prove, I like to come up with my own examples. Then I ask questions about the examples and try to figure out what I could say about them. It doesn't matter whether somebody else knows it already or not - I don't know how it works, so I'm going to do it and see what I come up with.

A lot of the time, the ideas behind the theorems I prove are super simple. And I think that's what a lot of math is - taking an idea that you have that's not very complicated, or it's not that interesting, but then using all this machinery to build the idea into a theorem or something that other people might be interested in. And that's kind of why you have to learn all these areas of math - you have to learn all of the machinery to build your ideas.

We know that we can get discouraged or burned out in areas of our lives. What do you do when you feel like you're burned out in math?

I mostly get burned out by administrative tasks rather than math, but it's certainly true that math can cause burnout too. The things that really help me are going to conferences, where I can see my friends and see interesting things that they're doing, and just being with other people. It's much more fun to do things when you're with people that share your goals and struggles.

What advice would you give to young people studying mathematics?

Consistent effort over time is very important. Waiting until the day before the test to learn the material

usually causes problems, so it's really important to just sit down with things a little bit, day by day so that you can really learn.

Having human resources is also key for mathematics. Being able to talk to someone - like friends, a professor, or students that have taken the class before - really helps you to not get too frustrated. I think frustration tends to cause people to tune out or give up, so it's really important to not get too frustrated. As long as you don't tune out or give up, you can surely do it. So in that way, for the purposes of staying positive, I think it is crucial to find friends and people to talk to.

Conclusion

Dr. Speegle has traveled the arduous road of scholarship, and he would tell you that his mathematical journey is far from over. Each of us, embarking on our own spiritual and intellectual journeys, even ones not related to mathematics, has something to learn from him. We learn from Dr. Speegle that math is hard for everyone - no matter who you are or what level of math you are at, it's hard to understand these beautiful, intricate concepts. And for that reason, there is always someone next to you who is just as confused as you are. Dr. Speegle teaches us to not be afraid of this seeming futility; in fact, he teaches us to embrace our confusion by tackling it with friends. He teaches us that it only takes one or two fellow students, friends, or instructors to make us really understand how confused we are - that it is easier than we think to be part of the mathematical community. Dr. Speegle teaches us to persevere - he tells us to sit down with our problems day by day, work at them, ask for help, and enjoy them with friends. How could it have been so simple? How could it be that the answer to our mathematical struggles is the exact same practice we wish to cultivate with everything in our lives? Every day, we work at ourselves - we struggle to find ourselves, we become confused, and we become discouraged. But we remember our community and the networks of support around us - we ask for help, and we rejoice with our friends at the accomplishment of things we previously thought impossible. Dr. Speegle teaches us that we have known the answers to our mathematical struggles all along, and we thank him for revealing such a powerful truth.

Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called "EvenQuads" to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.



Maryam Mirzakhani
(1977-2017)

Mirzakhani was from Iran and did her graduate studies in the US. She was a full professor at Stanford University until her untimely death from breast cancer, having previously been at Princeton University. Mirzakhani was awarded the Fields Medal (2014) for her work on hyperbolic and symplectic geometry. This made her both the first woman and the first Iranian to win this prestigious award. She was made a member of the National Academy of Sciences (2016), and that body now awards the Maryam Mirzakhani Prize in Mathematics in her honor. Mirzakhani's birthday, May 12th, is known as a day for celebrating women in mathematics.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.

EVENQUADS



Fan Chung
(b. 1949)

Chung is from Taiwan and did her graduate work in the US. She worked at Bell Labs and Bellcore for twenty years before moving to the University of California, San Diego. Her research is in graph theory and combinatorics, where she has collaborated with over 120 mathematicians and published 300 papers. Chung has served as editor-in-chief and on the editorial board for many journals. She was an invited speaker at the International Congress of Mathematicians (1994) and was the AWM Noether lecturer (2009). Chung received the MAA's Allendoerfer Award (1990) and Euler Medal (2017). She is a fellow of AAAS (2002), AMS (2013), and SIAM (2015).

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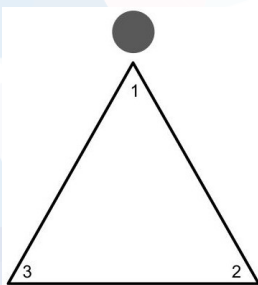
Games on Shapes

By Emily Twardy

Masters Student, Department of Mathematics and Statistics, Saint Louis University

Introduction

Allow me to teach you a game for two players. We start by selecting a regular polygon; for example, a triangle. Next, we label each vertex with a number such that Vertex 1 is at the top, and the numbers continue sequentially clockwise. Then we place a token or a mark on Vertex 1.



The players each take on one of two roles: The Mover and the Returner. The Mover and the Returner take turns using the *symmetries* of the triangle to move the token to different vertices. For the triangle, the symmetries (the possible moves) are three rotations - rotation by 0° (the identity), 120° , and 240° clockwise - and three flips, over Vertices 1, 2, and 3.

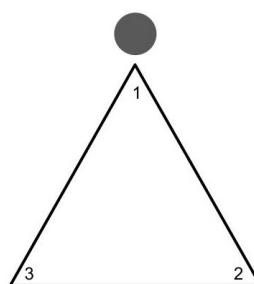
The Rules

Once we have the triangle and the symmetries of the triangle (i.e. the elements of the dihedral group), the game follows as such:

1. We put a token on Vertex 1.
2. We agree on which player goes first.
3. The first player chooses a symmetry, which may (or may not) move the token to a different vertex.
4. The second player chooses a different symmetry, and the token might move again.
5. Play repeats like this, with each player choosing a symmetry that has not already been selected, until there are no more symmetries left.

The Returner wins only if the token ends back on Vertex 1. The Mover wins only if the token ends up somewhere else.

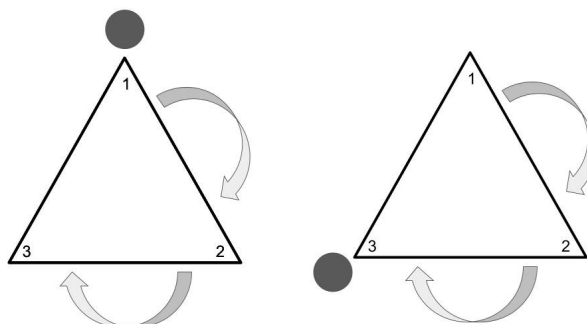
Example: The Triangle



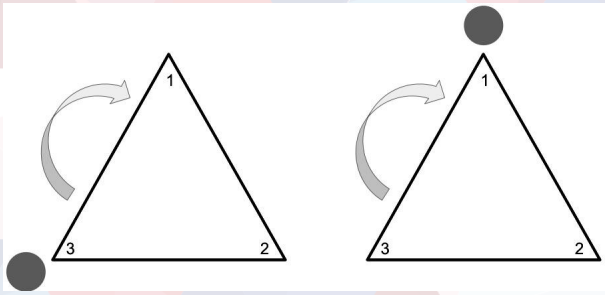
Let's see a few examples of how the game might be played. Then you'll be able to draw a triangle and play with a friend. The Returner and the Mover start by choosing which shape they want to use. For our first game, they choose the triangle and write up the list of possible moves. For the triangle, there are 6 moves to make: three rotations and three flips.

1. e , the identity, or the rotation by "zero degrees"
2. r^1 , the clockwise rotation by 120°
3. r^2 , the clockwise rotation by 240°
4. f_1 , the flip which switches 2 and 3 while fixing 1
5. f_2 , the flip which switches 1 and 3 while fixing 2
6. f_3 , the flip which switches 1 and 2 while fixing 3

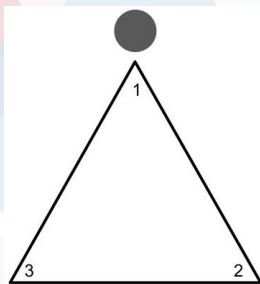
Now, they are ready to play. They agree that this time, the Returner will go first. The Returner might rotate the token around the triangle by playing r^2 , putting the token on Vertex 3.



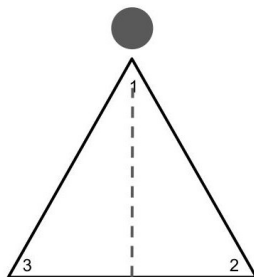
After that, the Mover rotates the token around the triangle, playing r^1 , putting the token back on Vertex 1.



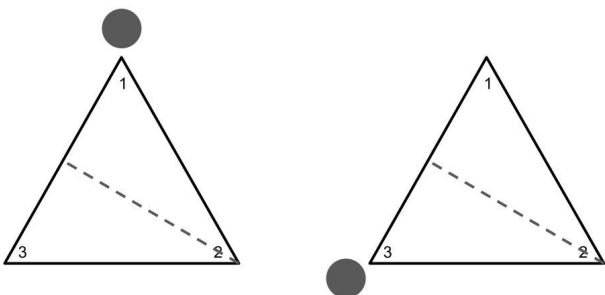
Note that the two rotations played are *inverses* of each other, meaning that one “undoes” the other so that the product of those two rotations is e , the identity. The Returner’s goal is to end the token on 1, so they select e , leaving the token on 1.



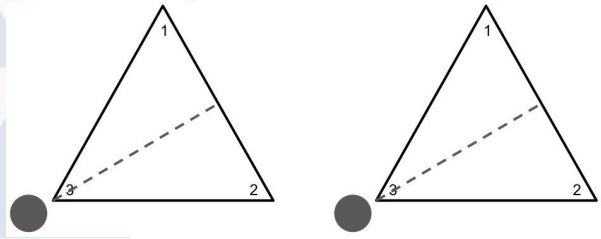
After that, the Mover selects f_1 , which still leaves the token on Vertex 1. The product of the last two moves is f_1 .



The Returner might then flip the token over Vertex 2, putting the token on Vertex 3.



The Mover only has one move left, the flip over 3, leaving the token on Vertex 3.



We are out of moves, and the token did not end on 1, so the Mover wins this time.

Questions

While the Mover won *this* game, is it possible for the Mover to make worse moves, and the Returner to make better moves, letting the Returner win?

Since there are no ties (either the token ends where it started or it does not), and the game has a finite number of moves, there is an ideal strategy, meaning that one player can create a strategy such that they will *always* be able to win. Which player can always win? What strategy might they employ to always win? Does it matter who goes first? In our first example, the Returner went first. Does our answer change if the Mover goes first?

What if instead of a triangle, we played the game on a shape with more symmetries, like a square or a pentagon? Do our answers change?

We can use ideas from group theory to prove that (regardless of who goes first) when playing on an n -sided polygon, the Mover will always win, *unless* n is even but not divisible by 4. For example, in the case of the hexagon, the Returner can always win.

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History of Mathematics: the Akhmim Wooden Tablets

By Anneke Bart

Associate Professor, Department of Mathematics and Statistics, Saint Louis University

The history of mathematics goes back a very long time. Ancient Egyptian mathematics and Babylonian mathematics provide some of the oldest examples of mathematics. Egyptian mathematical texts include the Rhind papyrus and the Moscow Mathematical Papyrus, both of which show lists of math problems with their solutions. The problems include both arithmetic problems and geometry problems.

Not as well-known as the Rhind and Moscow papyri are the Akhmim wooden tablets, which now reside in the Cairo Museum. These tablets date back some 4000 years and are early examples of work done by scribes to compute divisions of the measure called a hekat.

Ancient Egyptians measured volumes of beer, grain, and bread in terms of so-called hekats. The modern equivalent is approximately 4.8 liters. Dr. Annette Imhausen, a German historian who has done significant work in Ancient Egyptian mathematics, describes how mathematics was used to compute how one would distribute food among a certain number of individuals. Imagine, for instance, priests or soldiers sharing a certain amount of beer or bread.

The Akhmim wooden tablets are named after the city of Akhmim where they were found. The two wooden tablets, each measuring 18 by 10 inches, are covered in plaster and are inscribed on both sides. The tablets were first mentioned by an Egyptologist named Daressy in 1901, and he followed up with a paper describing the mathematics in 1906. The tablets were again described by T. Eric Peet in 1923. The most recent examination of the tablets and the mathematics they contain was done by Dr. Hana Vymazalova in 2002, [3].

The Akhmim wooden tablets originally contained a copy of a letter, which is now much destroyed. There are also two separate lists of names included. Most of the tablets, however, are covered with computations involving fractions.

Ancient Egyptians had an interesting system for

dealing with fractions. With the exception of the fraction $\frac{2}{3}$, they preferred writing all their fractions in the form of $\frac{1}{n}$. This meant that they would decompose any fraction into a sum of these unit fractions. They would not record a fraction of $\frac{3}{5}$, but would express that quantity as $\frac{1}{2} + \frac{1}{10}$.



Figure 1: Egyptian fractions were usually written as $1/n$. Most of the hieroglyphs were written as r (mouth hieroglyph) with dashes indicating the number n . The exceptions were $1/2$ and $2/3$ (Image by author)

Besides these unit fractions, there was also a set of fractions called the Horus eye fractions.

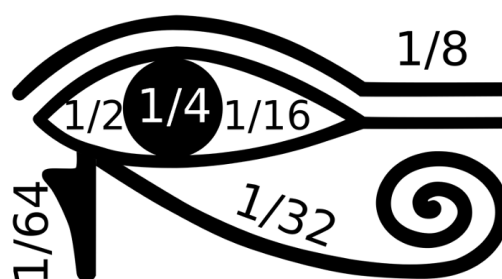


Figure 2: The Horus Eye with associated fractions. (Wikimedia commons: File: Oudjat.svg)

These fractions were all powers of $\frac{1}{2}$, and these fractions had special hieroglyphs. In some instances, these Horus eye fractions were preferred, and the scribes

would choose to write $\frac{1}{3}$ as

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64}$$

for instance. The scribes would know that the number 1 could be rewritten as

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + 5ro,$$

where $5ro = \frac{1}{64}$.

The so-called hekat problems are common in the extant mathematical papyri. An example from the Rhind mathematical papyrus (ca 1500 BC) discusses how to divide a hekat of grain into smaller units called henu.

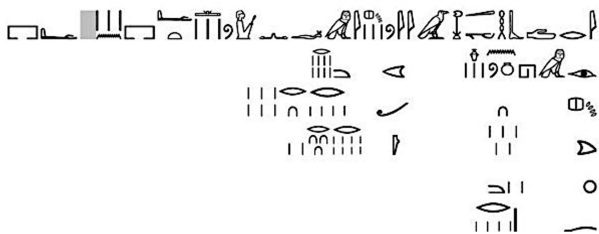


Figure 3: Transliteration of problem 80 from the Rhind Mathematical Papyrus. Note the hieroglyph for a barrel of grain (a hekat) on the third row on the right and the Horus Eye fractions directly below that. (Image by author)

The math problems on the Akhmim wooden tablets show how one writes $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{10}$, $\frac{1}{11}$, and $\frac{1}{13}$ in terms of Horus eye fractions. Furthermore, it shows how to check that the answer is correct. The calculations for $\frac{1}{3}$ of a hekat appear twice and show correctly that

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + (1 + \frac{2}{3})ro.$$

The scribe then proceeds to check the computations by showing that

$$3 * \left(\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + (1 + \frac{2}{3})ro \right) = 1.$$

An interesting situation arises in the $\frac{1}{7}$ computation. Here, the scribe makes the mistake of stating in the calculations that $2 * 7 = 12$ and $4 * 7 = 24$. This same mistake occurs in all of the four computations shown. Somehow, however, the scribe does arrive at the correct answer in the end.

Interestingly, the same mistake seems to be copied in each of the four exercises. The work shown gives the impression of a scribe who is practicing the process of performing these computations by copying (somewhat

mindlessly) the steps required to do the division and then multiplication to check one's work for correctness.

The Akhmim wooden tablets are just one of many examples of mathematics as practiced by the Ancient Egyptians a long time ago. Their treatment of fractions was very different from our own, as was their procedure for multiplying and dividing numbers. The mathematics problems that have been passed down to us show a practical approach to problems they faced in administering their religious centers as well as the country-wide building programs.

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Women Mathematicians Corner



Sylvia T. Bozeman
(b. 1947)

Bozeman is professor emerita at Spelman College, where she served for four decades. She is an analyst who has long been a strong advocate for building mathematical communities and increasing the participation of women and under-represented minority groups. Bozeman co-founded the Enhancing Diversity in Graduate Education (EDGE) Program (1998) to support women moving through graduate school in mathematics. Since then, over 250 women have participated in the program, leading to over 100

PhDs. She was the first Black American elected as an MAA Section Governor (1997), and is an AMS Fellow (2012) and an AWM Fellow (2018). Bozeman won the inaugural MAA Award for Inclusivity (2019).

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.

EVENQUADS



My Favorite Students

By Lauren Miller

Assistant Professor, Department of Mathematics and Statistics, Saint Louis University

I want to share with you the characteristics that make up my dream students; my favorite students. Now, I know we're not supposed to have favorites but be honest, we all do. My favorite students aren't the ones who get A's or know all the right answers. What I look for are students who show up, pay attention, participate, question me, and push me to be a better educator. I am excited to share that I have found my dream students and have been working with them for almost 3 years. They may be a little different from what you're expecting. My favorite students are incarcerated.

It was a grey January morning when I arrived at the prison. The moisture in the air brought the cold deeper and deeper into my bones. Was I shivering or was I shaking? I was cold, I was nervous, and I was a little scared. I was willingly entering a place no one wanted to be, and my first and largest fear was that I would not be let in. Had I followed all the rules to make it through security? Did my outfit pass the dress code? I approach the front door under the watchful eye of the guard tower. I walk in, and I am scanned and searched. I sign in, scan my ID, show my ID, am handed a body alarm, affectionately called a screamer, scan my ID again and enter the airlock, show my ID for the final time, exit the airlock, and enter the yard. About an acre and a half of dry brittle grass lay before me, with snaking sidewalks that appear to have been an afterthought. To the left of me are four long cream brick houses with tiny, barred windows, each separated by barbed wire, with a guard stationed outside each one. Ahead is a long building that looks like an abandoned strip mall containing the cafeteria, laundry, GED school, maintenance, and my final destination, the education wing.

I enter the classroom under close watch; there is a CO, a correctional officer, sitting in the next room, watching me on a camera aimed directly at the whiteboard. I set up my PowerPoint and got ready to introduce myself and learn twenty-one new names as quickly as possible. It was the first day of a new semester, and I was so excited to meet my students, but I was more nervous than excited this time. This semester was different; these students were different.

But it would take me time to realize how truly not just different - but remarkable - they were.

Every hour, on the hour, is movement: the only time incarcerated persons can move around the prison. At 8 A.M., my students came boisterously rushing in. Twenty-one men aged 26 to 68, all dressed in khaki uniforms given to them by the DOC, excited to meet a new professor, were all ready to learn with a college algebra textbook in hand. They took their seats and opened their notebooks, and the second I started speaking, they fell silent. Once I met them and saw their faces and their smiles, the nerves were gone, and only excitement remained - excitement to share something I love with others. Since then, this population of students has become my favorite. My favorite students are incarcerated.

My favorite students are pursuing bachelor's degrees and associate degrees. They want to be computer scientists, poets, engineers, and Ph.D. astronomers. They have plans to save the planet, revitalize their neighborhoods, and inspire others. My favorite students are incarcerated.

In prison, my classroom is vibrant with voices sharing connections I had never made before. Everyone has a question they are not afraid to ask - everyone has an answer and will shout it confidently even if they're wrong. Outside of prison, I teach some of the most privileged students in America. My favorite students are incarcerated.

In prison, my class is made up entirely of nontraditional students. Some have been in prison since before I was born. Many did not complete high school and got their GEDs while incarcerated. This is where I can relate. I did not graduate high school. I dropped out in my junior year. I received my GED when I was nineteen. It took me another three years to enroll in college full-time. I did not know it at the time, but I would have been considered a nontraditional student. Now, a high school dropout and nontraditional student who was finishing her doctorate was teaching a class of entirely nontraditional students. This was January of 2020, and we all know what happened next.

Billiken Challenge

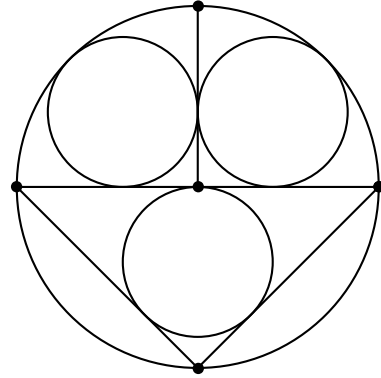
In March of 2020, universities shut down and moved classes online, and so too was the case for my incarcerated students. However, they don't have access to the internet. They have tablets that are easily broken, with little storage for video lectures. Prison educators rallied as all educators did when schools shut down. We just had another barrier - one of a lack of resources and access. We were remote via our learning management system for the rest of that semester, and then the summer, the fall, and the spring again. My students took finite math, statistics, and discrete structures. All asynchronous, and all online.

In the Summer of 2021, I was allowed back into prison for the first time in over a year. I was able to see my students again. I often get teased that I smile too much when I am in the prison. I'm just happy to see some of my favorite people. Since that first course, I have expanded my reach in prison education. In the Spring of 2022, for the first time, I taught in a women's prison. This was the inaugural class, meaning now 100% of women's prisons in Missouri have access to quality in-person higher education. This Fall, I was able to teach the first course that brought me into prison education again: College Algebra. I even had a TA - a former student from my Spring 2020 class.

Since that first college algebra course, many of my students have graduated. Some of my students have been released. I have met them for coffee, and I am able to watch them adapt to their new life and take their knowledge with them. Many of my students have a release date set, so they know when they are getting out. I imagine they are a mixture of excited and nervous to enter an outside world that changed around them, but they have changed too. Horace Mann said that "Education, then, beyond all other devices of human origin, is the great equalizer of the conditions of men." This has never been truer than for my justice-impacted students. Currently, there are over 380 prison education programs in the U.S. that support associate's through master's degrees for incarcerated men and women. The work isn't for everyone, but if you would like to get involved, I would be happy to introduce you to my favorite students.

Circle Construct

Place four equally spaced points on a circle of radius one. The picture shows the four points, and the circle's center is marked with dots.

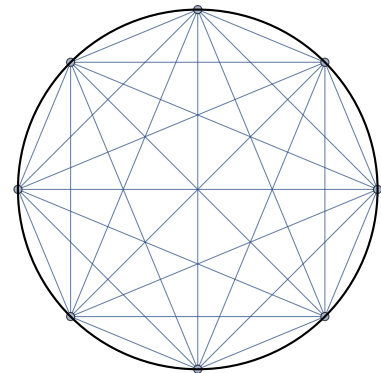


Show that the three small circles all have the same radius.

For solutions, see our website at <http://mathstat.slu.edu/about/newsletter>

Octagon Chords

Place eight equally spaced points on a circle of radius one. Connect every pair of points with a chord.



What is the product of the lengths of all the chords? Can you generalize to n equally spaced points?

For solutions, see our website at <http://mathstat.slu.edu/about/newsletter>

Is There a Fractional Dimension?

By Elodie Pozzi

Assistant Professor, Department of Mathematics and Statistics, Saint Louis University

A natural response is “No.” But when answering this question, we may be thinking of the innate concept of dimension; in that case, there does not exist any object of fractional dimension for the simple reason that the dimension is always an integer. However, one could define another concept of dimension for which some geometric objects will have a fractional dimension. But why define a new concept of dimension? In what follows, we answer this question and the question from the title.

Which dimension

From our very early childhood, we have been aware of shape and space. Indeed, humans need to develop basic spatial concepts to live adequately in the real world. We are thus able to perceive the difference between triangles and pyramids, squares and cubes, for instance. When talking about a geometric shape in the plane or in space, we can talk about its border or the filled-in object. Precisely, we say a full (or filled-in) square in reference to the contents of the square and its border, and we say a filled-in sphere (or ball) to reference the sphere and what is inside the sphere (see Figure 1).

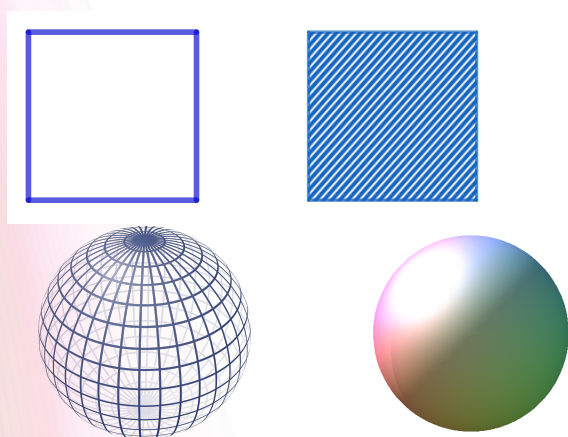


Figure 1: From left to right: Square, filled-in square, sphere and ball (first, second and last picture drawn with Geogebra; third picture from Wikipedia)

The notion of dimension can be found everywhere: in sciences but also in art and literature. Beyond the innate perception we have of dimension, let us give a first definition:

Definition The dimension of a geometric object is its degree of freedom.

Roughly speaking, to draw a line, one needs a point and a direction; thus, its dimension is one. For a filled-in square, one needs a point and two directions. From the mathematical point of view, the degree of freedom corresponds to the number of parameters needed in the parametric equation of the geometric shape: one parameter for a curve $(x(t), y(t))$, t being real, and two for a surface $(x(u, v), y(u, v), z(u, v))$, with u and v being real parameters.

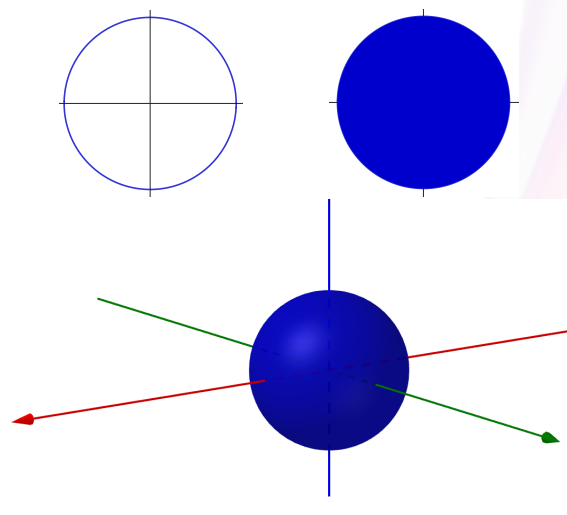


Figure 2: Circle: $(\cos(t), \sin(t))$, $0 \leq t < 2\pi$, disk: $(r \cos(t), r \sin(t))$, $0 \leq r < 1$, $0 \leq t < 2\pi$ and sphere of radius one $(\cos(\phi) \sin(\theta), \sin(\phi) \sin(\theta), \cos(\theta))$, $0 \leq \phi < 2\pi$, $0 \leq \theta \leq \pi$, drawn using Geogebra.

As a consequence, a line has dimension one, a full square has dimension two, and a ball has dimension three. Note that these dimensions are all integers. This notion of dimension is not only applicable to mathematical shapes but also to shapes that appear in nature. As an example, if we apply the definition to the brain,

the dimension of the (filled-in) brain is three, and the border (or surface) of the brain has dimension two.

But if we take a close look at the shape of the brain and compare it to a ball, the structure of the brain looks more complex, even if the dimensions are the same. This means that this dimension is not subtle enough to distinguish the geometric properties of the two shapes. Actually, the concept of dimension we have learned from our childhood is called the *topological dimension*. The word topological refers to what remains unchanged under continuous deformations. If one deforms a curve in a continuous way (*i.e.* shrinking or dilating without tearing it), the dimension does not change; the curve will still have one dimension even though its appearance has changed.

However, it is possible to define a new concept of dimension that will capture differences in geometric complexity. The question is just how to do it. While the topological dimension considers a curve, or more generally, an object in its whole, *i.e.* globally, the new dimension should be computed following the different aspects of the geometric object. To that purpose, we will cover a figure in the plane using squares of side length r . As a first example, we consider a line segment of length 1 (see Figure 3). Of course, a line segment is not geometrically complex, but it gives us a first idea of the covering process. Let us denote by $N(r)$ the number of squares of side length r needed to recover the line segment.

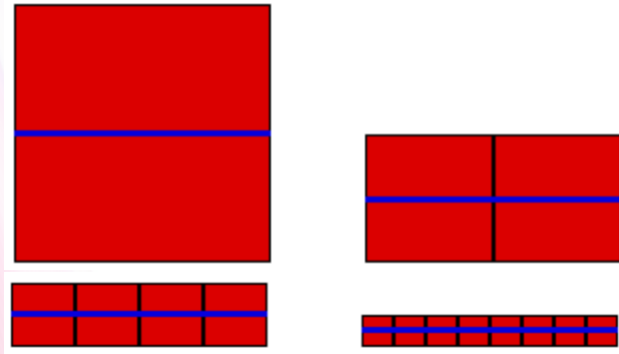


Figure 3: Box-counting of the line segment of length 1, modified from [3].

It is clear that the smaller r is, the larger $N(r)$ becomes. More precisely, the length of the line segment $\ell = 1$ is equal to $N(r)r$. Then Figure 3 shows that we obtain the formula $N(r) = \frac{1}{r}$. Let us proceed in the same way to fill-in a square of side-length 1. The number of squares of side-length r needed to cover the filled-in square is $N(r) = \frac{1}{r^2}$. For a cube of side-length 1, the number $N(r)$ of cubes of side-length r is $\frac{1}{r^3}$ as shown in Figures 4 and 5.

From the expression of $N(r)$ in the situations de-

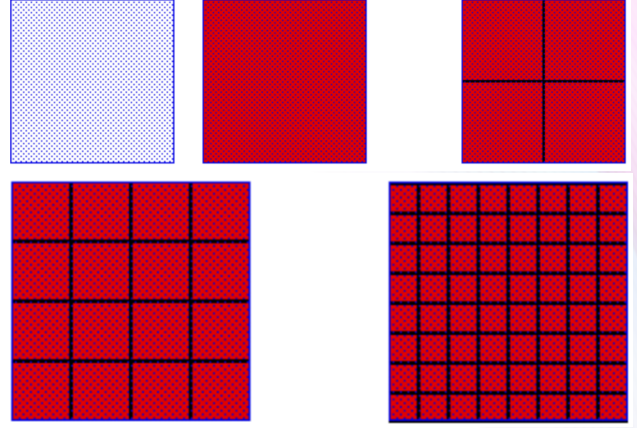


Figure 4: Box-counting of the filled-in square of side-length 1, modified from [3].

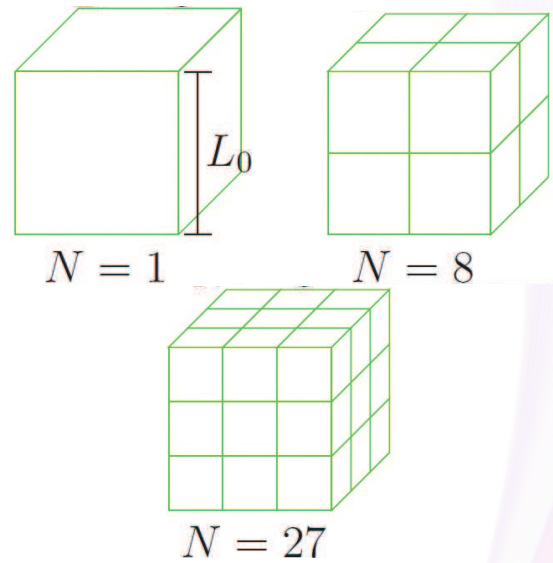


Figure 5: Box-counting of the filled-in cube of side-length L_0 , modified from [9].

scribed in Figures 3, 4 and 5, one has

- $N(r) = \frac{\ell}{r}$ for a line segment of length ℓ ;
- $N(r) = \left(\frac{\ell}{r}\right)^2$ for a square of side-length ℓ ;
- $N(r) = \left(\frac{\ell}{r}\right)^3$ for a cube of side-length ℓ .

If we apply the logarithm $\ln$ on both sides of the equation giving the value of $N(r)$, we get $\ln(N(r)) = d \ln(\ell) + d \ln(1/r)$, and

$$\lim_{r \rightarrow 0^+} \frac{\ln(N(r))}{\ln(1/r)} = \lim_{r \rightarrow 0^+} d \frac{\ln(\ell)}{\ln(1/r)} + d = d,$$

with $d = 1$ for a line segment, $d = 2$ for a square and $d = 3$ for a cube. In each case, d corresponds to

the topological dimension of the geometric object. The value d is called the **box-dimension** and is defined by

$$\text{box-dimension} := d = \lim_{r \rightarrow 0^+} \frac{\ln(N(r))}{\ln(1/r)}. \quad (1)$$

In other words, for simple geometric shapes like a line segment, a square, or a cube, the box dimension coincides with the topological dimension. What happens when the geometry is more complex? Let us do the same process with a more complex curve: the *von Koch curve*. This special curve is the result of a recursive process starting from a line segment denoted by S_0 . At step 1, we delete the middle third of S_0 and replace it with the other two sides of an equilateral triangle. The next steps are generated recursively by the same process: S_n is obtained by replacing the middle third of each line segment in S_{n-1} by the other two sides of an equilateral triangle. The von Koch curve is the limit as n goes to $+\infty$: $\lim_{n \rightarrow +\infty} S_n = K$, see Figure 6.

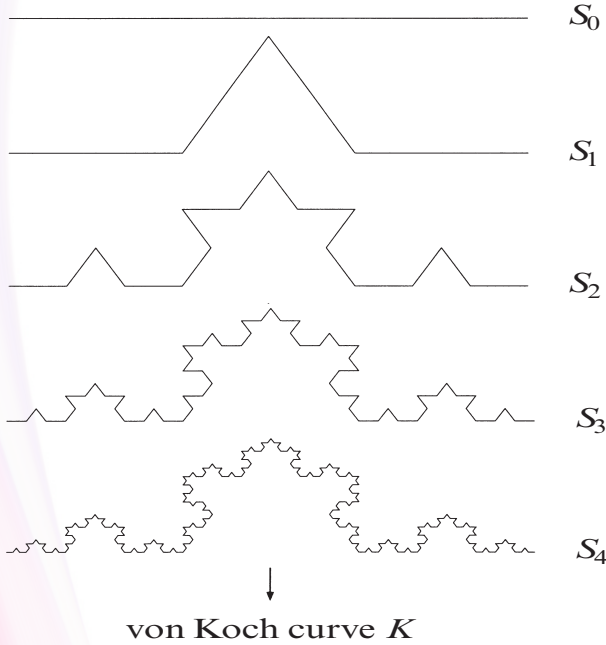


Figure 6: First four steps of the construction of the von Koch curve, from [12].

We cover the von Koch curve using squares of side-length r . We assume that $\ell_0 = 1$ is the length of S_0 .

The different pictures in Figure 7 illustrate different ways to cover the von Koch curve by squares of side-length r . Table 1 shows the values of $\ln(N(r))/\ln(1/r)$ for the different values of r corresponding to Figure 7 generated by a computer.

This method gives an approximation of the box-dimension d of the von Koch curve and can be applied to other curves or three-dimensional shapes. The von Koch curve is known for being *self-similar*, which means

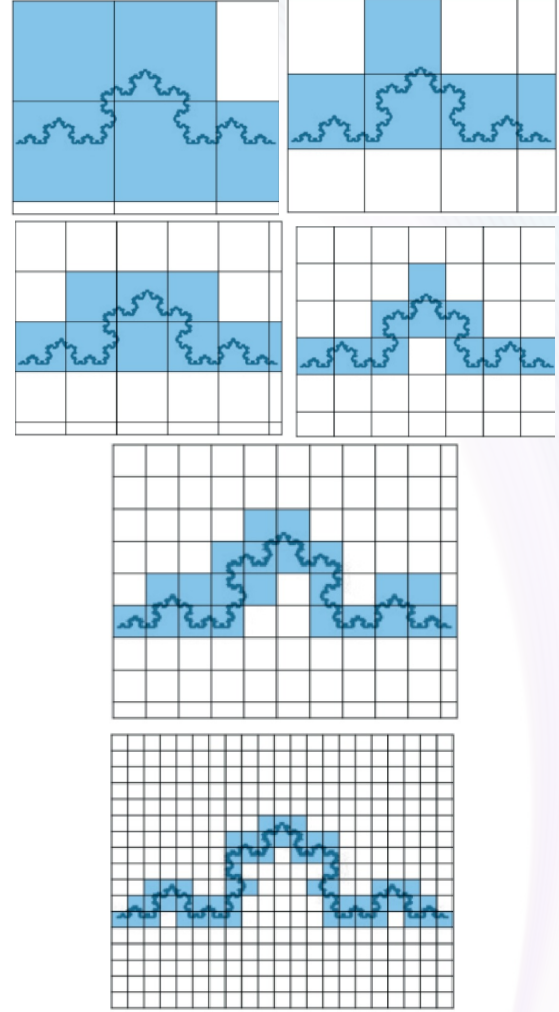


Figure 7: Box-counting method applied to the Koch curve, from [5].

r	$N(r)$	$\ln(N(r))/\ln(1/r)$
$1/3$	4.5	1.36
$1/(3.2)$	4.24	1.24
$1/(5.1)$	8.1	1.28
$1/7$	10	1.18
$1/(10.1)$	21.1	1.29
$1/21$	56	1.31

Table 1: Table of estimations of $\ln(N(r))/\ln(1/r)$

that it is similar to itself at all scales as shown in Figure 8.

Thanks to this special structure, it is possible to have an exact value of d : the curve has four equal pieces, scaled by a factor of $1/3$ in both directions. We thus need four squares of side-length $1/3$ to cover the curve. More generally, one can cover the curve using 4^n squares of side-length $1/3^n$ for $n \geq 1$. We deduce that

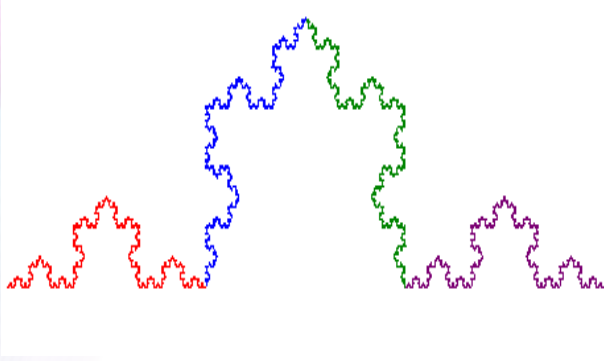


Figure 8: Self-similarity of the von Koch curve from [11]

$$\begin{aligned}
 d &= \lim_{n \rightarrow +\infty} \frac{\ln(N(1/3^n))}{\ln(3^n)} \\
 &= \lim_{n \rightarrow +\infty} \frac{\ln(4^n)}{\ln(3^n)} \\
 &= \lim_{n \rightarrow +\infty} \frac{n \ln(4)}{n \ln(3)} \\
 &= \frac{\ln(4)}{\ln(3)} \approx 1.26 > 1 = \text{topological dimension.}
 \end{aligned}$$

In those lines of computations, r is replaced by $\frac{1}{3^n}$ for $n \geq 1$. Actually, it is possible to change r by a sequence of values that converges to 0 in the definition of d given in Equation (1). The self-similarity property allows computation of the exact value of d . When the curve is not self-similar, we estimate $\ln(N(r))/\ln(1/r)$ by using a log – log plot from a table of values comparable to Table 1. Using the example of the von Koch curve with Table 1, we place the points $(\ln(1/r), \ln(N(r)))$ for different values of r :

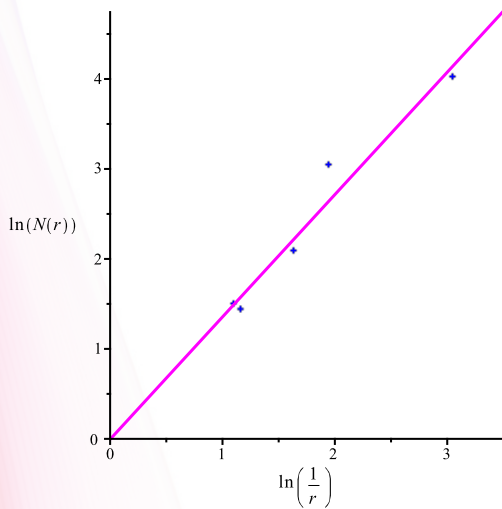


Figure 9: log – log plot

The estimate can be more precise using computer code; the precision depends on the method the code

uses to cover with squares. Among the examples seen above, some have a box dimension equal to the topological dimension, while others have a box dimension strictly greater than the topological dimension. This observation allows us to distinguish two kinds of geometric shapes/objects S :

Group 1: The topological dimension of S is strictly smaller than the box-dimension;

Group 2: The topological dimension of S is equal to the box-dimension.

Observe that there is no geometric shape with a topological dimension strictly greater than the box dimension. The geometric shapes that belong to Group 1 are called fractals, and the ones that belong to Group 2 are called non-fractals. There exists a more general definition of dimension using shapes that are not restricted to squares or cubes which will be more adapted to the complex aspect of the object S . We will not give the explicit definition; interested readers can refer to [10] or [13] for more details. Interestingly, it is possible to find fractal shapes in nature with some estimates of the box dimension (see Figure 10).



Figure 10: Tree (box-dimension ≈ 1.8), the coastline of Ireland (box-dimension ≈ 1.22); Self-similar river network from the Shaanxi province in China (box-dimension ≈ 1.9), pictures from [8], [4] and [2].

The box dimension is a mathematical concept that measures and classifies the complexity of any geometric shape and, in particular, the geometric shapes found in nature. Most of the previous examples had a topological dimension equal to two. Now, let us consider the three-dimensional case: there are computer codes able to give an estimate of the box dimension of surfaces like broccoli, the human brain, and the human lungs. To get a precise estimate of the box dimension of surfaces, the computer codes become more elaborated. We give below an estimate of the box dimension for the surface of broccoli, the human brain, and human lungs (see Figure 11).

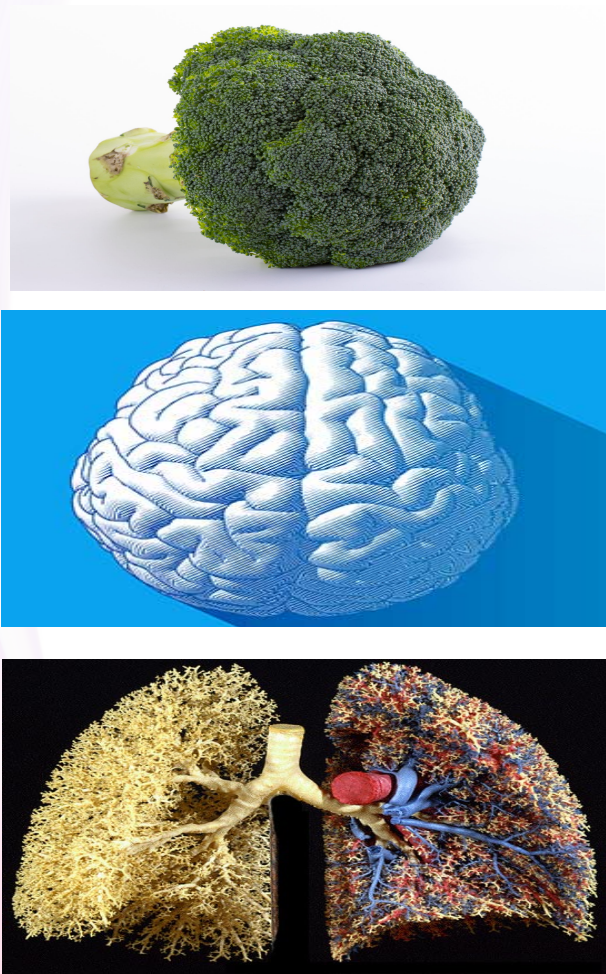


Figure 11: Surface of: broccoli (box-dimension ≈ 2.7), a human brain (box-dimension ≈ 2.8), human lungs (box-dimension ≈ 2.97), pictures from [7], [6] and [1].

To conclude, the box dimension gives a subtle classification of geometric shapes. From the mathematical point of view, the classical analysis on the real line $\mathbb{R}$ or in the plane or in space (taught in Calculus I, II and III classes) is not fine enough to be used on fractals. However, mathematicians have developed and continue to expand analysis on fractal domains.

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The Integration of Math and Music

By Maram Lemnifi

Undergraduate Student, Saint Louis University

While many people may think that math and music have nothing in common, studies have shown that there is a significant correlation between the two. Math itself is known to be applicable to many real-world scenarios, whether it be baking, building, or finances. Music is a way of leisure or craft that is made to be enjoyed and heard. However, there are many mathematical applications that are involved in making music sound enjoyable. The pleasure found in listening to music would not be the same if not for math. For example, math is used to time the spacing of music to create rhythm or to sound more synchronized. The same can also be applied the other way around, with music having an impact on learning or academic performance. Research has been done on how music can sometimes enhance academic performance or the ability to focus. Already, it is evident that music and math are both deeply integrated and dependent on one another, enhancing each other's capabilities and affecting us as human beings in our daily lives.

Ever wondered why music is so pleasing to the ear? The repetitive patterns within a song or piece are what attract people to listen. According to the Santa Fe Institute, both math and music are considered to be symmetrical. Symmetry is usually defined as when one half of a specific element or body is the same as the other half. In this case, imagine specific sounds within a musical piece played and then repeated again either by being spaced out within seconds of each other or possibly within different halves or parts of the musical piece. There are many other instances where symmetry can also be found, such as in nature. The human body can be seen as symmetrical, as well as the bodies of other animals that live among us. For example, it may look out of place if we or other creatures did not have one eye to match the other on both sides of our face, or if we didn't have two legs or two arms. These patterns and symmetrical observances are pleasing both to the human eye and ear. They allow for a sort of peace of mind when seen and heard - a feeling of contentment or satisfaction.

Music is also seen as a type of sound. Sounds are waves that travel and vibrate throughout the air until they reach your ear, and it is then that you begin

to hear. The way that wave travels within the air affects our auditory perception of it. The frequency of a wave is the number of times one wave, cycle, or vibration occurs per second. The higher the frequency, the more cycles occur within the one-second interval. This would equate to a quicker and more-pitched sound. With a lower frequency, the sound would be slower and less high-pitched. These mathematical and physical concepts behind sound are all ways in which music functions and these are all things that musicians or builders have to think about regarding concert halls. The walls of a concert hall are specifically designed to bounce sound off in a specific way so that the best acoustics possible can reach the ears of the audience. Typically, soft surfaces absorb sound waves, while hard surfaces reflect them. Therefore, the only soft surface found in concert halls is the seating [3]. Builders of a concert hall also must make sure that no other sound from outside the hall can be heard, such as traffic or city noise. They can do this by surrounding the hall with air, meaning that the walls would be thick and insulated, helping with the overall vibration of sound [3].

In addition, a correlation between music and the learning efficiency of children has also been shown in many studies. According to a study done by James Didimenico, it was found that many elementary school teachers used music as a method of helping students learn in the classroom. They would use it to "alter the mood" in the class or keep students on task and focused. Students would also begin to associate certain sounds in the classroom with specific meanings. For example, one of the first-grade teachers evaluated in the study would use chimes to notify the students when to stop, clean, and transition to a different activity. Music was also a way for some teachers to help students learn the subject material. It was said that when the subject or topic was a bit "dry", teachers would use shows or videos from Schoolhouse Rock or GoNoodle, both of which are music-based educational organizations, to help the students feel more active and focused while learning [4].

Both math and music also require consistent practice to be successful. Math is not something that an individual can be good at automatically

without understanding its rules, laws, or methods of solving. It is something that requires time and thought. The same can be applied to music in relation to many different types of musicians, be they singers or classical musicians, practicing for hours every day to perfect their craft. For example, it is well known that professional pianists can spend up to 8-10 hours practicing per day [7]. However, according to Psychology Today, a study was done to see if students who practiced more ended up with better results than those who didn't practice as much. It was found that the number of hours practiced did not equate to better performance. Instead, it was those who took the time to practice and analyze the harder passages who did well. It was the quality of their practice and the study method behind the practice that resulted in a much better performance [2]. All these concepts can be applied to the students of math as well.

In conclusion, these insights into how math and music are correlated in our world are only a select few and do not even begin to scratch the surface of their intricacies. Much is still to be discovered about the relationship between the two and their importance to humans and the world.

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Women Mathematicians Corner

The Association of Women in Mathematics (AWM) started a project called "EvenQuads" to celebrate their fiftieth anniversary by designing a playing deck of cards and honoring notable women mathematicians. Thanks to AWM for permitting us to share these playing cards in our magazine.



Hortensia Soto
(b. 1960s)

Soto was born in Mexico and grew up in western Nebraska. She is professor at Colorado State University, having previously taught at the University of Southern Colorado, and at the University of Northern Colorado. Her research and extensive publications center on the teaching and learning of undergraduate mathematics, with an eye towards equity in the classroom. Soto co-founded and directed the program Las Chicas de Matemáticas, an outreach program for mathematically talented high school girls, many of whom are headed towards being first-generation college students. Soto received an MAA Meritorious Service Award (2012) and an MAA Haimo Award (2018).

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

If you want to learn more about EvenQuads, scan the QR code below.

EVENQUADS



Students' Journeys

By Wedad Ali Alharbi

Doctoral Student, Department of Mathematics and Statistics, Saint Louis University

It all started with my first multiplication. I had already learned addition and subtraction, and my teacher introduced me to a new way of manipulating numbers. I was six years old; pretending that it was at that exact moment that I decided to study pure mathematics would be a lie.

Throughout my school years, I have appreciated mathematics and economics because I found they were complementary. That was until I was introduced to a mathematical theory about how intuition is actually an unconscious calculation of probability. This highlighted the true beauty of mathematics - its ability to be rational and abstract at the same time. What I particularly appreciate in this subject is the constant quest for answers and the challenges our brain has to put up with to solve a problem.

When a solution is found, it is undeniable, and mathematical laws prove it. This makes mathematical truths undebatable, whereas other subjects are always subject to questioning. I do not claim to possess a scientific mind, but I hope, with advanced studies in the mathematical field, I will acquire a logical, disciplined, and rational mind. That's why I chose to do advanced mathematics; to me, exact sciences are used to that effect - unraveling the mysteries of our universe and mathematically answering philosophical questions.

I have chosen to study in the United States since it offers the ideal balance among past, present, and future, for it blends traditional values with the evolution and progress of society. My Saudi, Arabic, and Muslim heritage have opened my mind to other cultures while teaching the importance of traditional values which protect and preserve our society.

"A woman cannot become a scientist." I have heard these rumors in my community on numerous occasions, and while my parents encouraged me in a wide variety of endeavors with the aim of raising a well-rounded individual, there was one area they did not ever encourage me to explore: my community strongly believed that science and, particularly, mathematics is not for women.

As my country is now leading a transformational movement, women have participated effectively in its development. Women have been assigned to multiple leadership positions, where they have given great, productive input into the new vision of Saudi Arabia.

I hope that by joining a US university, I will be able to complete my dreams and put them into faith. I will come back to Saudi Arabia to work as an assistant professor at my local university, and I will start to highlight the importance of mathematics and prove to my community the capabilities of women when they believe in themselves.

Women Mathematicians Corner



Raman Parimala
(b. 1948)

Raman Parimala was born in India and earned her PhD from Bombay University (1976). She is currently a Distinguished Professor at Emory University. Her research is in quadratic forms, Galois cohomology, and algebraic groups. She has published over 100 papers and advised 10 PhD theses. She was an invited speaker at the International Congress of Mathematicians (ICM) in 1994 and 2010 and gave the AWM-AMS Emmy Noether Lecture at the Joint Mathematics Meetings in 2013. Parimala is a fellow of all three Indian academies of science and of the AMS (2013). In 2020 she became one of 11 Indian female scientists, and the only mathematician, to have a chair at an Indian institution named in their honor.

This poster is part of the EvenQuads project, which combines innovative mathematical card games with learning about amazing women mathematicians. The project celebrates the 50th anniversary of the Association for Women in Mathematics.

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EVENQUADS



Normal Forms For Polynomial Dynamical Systems

By Benjamin Hutz

Associate Professor, Department of Mathematics and Statistics, Saint Louis University

In this article, we discuss a question about polynomial functions. Informally, a polynomial is a single variable function whose denominator is constant, e.g.,

$$f(x) = x^2 + 1.$$

The problem is to find a change of variables that represents all degree d polynomials in a “nice” form. For example, given that f and g have identical properties, which polynomial would you rather work with?

$$f(x) = x^3 + 1 \quad \text{or} \quad g(x) = 9x^3 + 9x^2 + 3x + \frac{1}{3}.$$

Of, course f is much simpler but given g , how you would find a change of variables that let you re-write it as f ? We present a solution for degree two and degree three polynomials. However, the problem remains open for higher degrees.

To understand the problem more precisely, we need to introduce the notion of a dynamical system.

Definition A dynamical system is made up of two pieces of information, a set S and a map $f : S \rightarrow S$.

Since f maps the set S to itself, we can *iterate*, i.e., repeatedly apply the function f . For readability, we typically denote iteration as an exponent:

$$f^3(t) = f(f(f(t)))$$

and, more generally,

$$f^n(t) = f(f^{n-1}(t)), \quad n \geq 2.$$

For a fixed $t \in S$, the *orbit* of t is the set of forward images

$$\{t, f(t), f(f(t)), f(f(f(t))), \dots\} = \{f^n(t) \mid n \geq 0\}.$$

Example Consider the function $f(x) = x^2$ acting on the set of rational numbers $\mathbb{Q}$. Then the orbit of 2 is given by

$$2 \xrightarrow{f} 4 \xrightarrow{f} 16 \xrightarrow{f} 256 \xrightarrow{f} \dots$$

These values in this orbit simply keep growing and the orbit contains infinitely many distinct points. However, the orbit of -1 is quite different; it contains only two distinct values:

$$-1 \xrightarrow{f} 1 \xrightarrow{f} 1 \xrightarrow{f} 1 \xrightarrow{f} \dots$$

There are two broad classes of problems one can study related to dynamical systems.

- (1) Fix a function f and study the properties of points under iteration of f .
- (2) Study the set of all dynamical systems arising from functions f that satisfy a certain property.

We examine the second problem in this article. Specifically, we want to examine a set of all dynamical systems arising from polynomials of some fixed degree.

We are looking for “nice” forms representing polynomials, so the first issue we need to address is when two polynomials have the same dynamical properties, i.e., when can we consider two polynomials equivalent?

Definition A linear fractional transformation is a function of the form $\alpha(x) = \frac{ax+b}{cx+d}$ with $ad - bc \neq 0$.

The restriction $ad - bc \neq 0$ is necessary so that the inverse α^{-1} exists. Recall that the inverse function satisfies

$$\alpha^{-1}(\alpha(x)) = x$$

We define the *conjugate* of a function f by a linear fractional transformation α as the composition

$$f^\alpha = \alpha^{-1} \circ f \circ \alpha.$$

Conjugation is the dynamical system equivalent of “change of variables.”

Example For the function $f(x) = x^2$ and the transformation $\alpha(x) = x + 2$, we have $\alpha^{-1}(x) = x - 2$ so that

$$\begin{aligned} f^\alpha &= \alpha^{-1}(f(\alpha(x))) = \alpha^{-1}(f(x+2)) \\ &= f(x+2) - 2 = (x+2)^2 - 2 = x^2 + 4x + 2. \end{aligned}$$

The reason conjugation preserves dynamical properties can be summarized as

The conjugate of the iterate is the iterate of the conjugate.

Specifically,

$$\begin{aligned} f^\alpha \circ f^\alpha &= \alpha^{-1} \circ f \circ \alpha \circ \alpha^{-1} \circ f \circ \alpha \\ &= \alpha^{-1} \circ f \circ f \circ \alpha = (f^2)^\alpha \end{aligned}$$

and, more generally,

$$(f^\alpha)^n = (f^n)^\alpha.$$

In fancier language, we would say that iteration and conjugation commute. This general premise is worth repeating since it is fundamental to our problem:

Conjugation preserves the dynamics.

So f and f^α have the same dynamical properties. For example, if an orbit contains only finitely many distinct values for f , the orbit of the conjugate also contains only finitely many distinct values.

We are almost ready to state our fundamental problem more precisely, but first, we need a change in perspective. So far, we have considered starting with a polynomial f and finding a change of variables (conjugation) so that f^α is “nice.” Depending on what you choose “nice” to mean, there are various algorithms that produce a simplified form for f ; for example, minimal models [3] or reduced forms [4]. The change in perspective is not to work one polynomial at a time, but rather to ask for a parameterized family already in a **nice form** that gives you every possible **distinct** polynomial dynamical system. The bold parts of the sentence represent the challenge. Since “nice form” depends somewhat on the application in mind, we will look mainly at “distinct”. Every polynomial of degree d can be written as

$$f(x) = a_d x^d + a_{d-1} x^{d-1} + \cdots + a_1 x + a_0.$$

However, many of these polynomials create the same dynamical system since they are conjugate. We already saw the following degree three examples of conjugate polynomials

$$f(x) = x^3 + 1 \quad \text{or} \quad g(x) = 9x^3 + 9x^2 + 3x + \frac{1}{3}.$$

Now we can state our fundamental problem more precisely.

Fundamental Problem. Fix a positive integer $d \geq 2$. Find a family of degree d polynomials $f_t(x)$ so that every polynomial is conjugate to some member of that family. Specifically, for any polynomial g of degree d , there exists parameter value t_0 and a linear fraction transformation α so that

$$g^\alpha(x) = f_{t_0}(x).$$

In fancy language, we are looking for representatives of the conjugacy classes in the moduli space of polynomial dynamical systems.

1 Degree Two Polynomials

Let’s see if we can solve the Fundamental Problem for degree two polynomials. Consider an arbitrary degree two polynomial

$$F = Ax^2 + Bx + C \quad \text{with} \quad A, B, C \in \mathbb{Q} \text{ and } A \neq 0.$$

An arbitrary conjugation by $\alpha(x) = \frac{sx+t}{ux+v}$ only sends polynomials to polynomials if $u = 0$. So we only need to consider conjugation by elements of the form $\alpha(x) = sx + t$. Computing the conjugation symbolically, we see

$$\alpha^{-1} \circ F \circ \alpha = Asx^2 + (2At + B)x + \frac{At^2 + Bt + C - t}{s}.$$

Since we can choose s, t arbitrarily, we can make two of the resulting coefficients anything we want. Specifically, choose $s = \frac{1}{A}$ and $t = -\frac{B}{2A}$. Then we have

$$\alpha^{-1} \circ F \circ \alpha = x^2 + \frac{-4B^2 + 16AC + 8B}{16A}.$$

In particular, we have shown the following statement.

Theorem 1. Every degree of two polynomials is conjugate to a polynomial of the form $x^2 + c$ for some constant c .

Theorem 1 solves both parts of the fundamental problem: we have a “nice” form for degree two polynomials that represent distinct dynamical systems. We still need to check the distinctness part. Specifically, for two different constants c and d , we need to show that $f = x^2 + c$ and $g = x^2 + d$ are not conjugate. Starting with f and conjugating again by the arbitrary $\alpha(x) = sx + t$, we get

$$\alpha \circ f \circ \alpha^{-1} = sx^2 + 2tx + \frac{c + t^2 - t}{s}.$$

So we must have $s = 1$ and $t = 0$ to get something in the right form, but then we have $\alpha(x) = x$ and

$$\alpha \circ f \circ \alpha^{-1} = f.$$

Specifically, f and g are not conjugate. We can then improve Theorem 1 to say the following.

Theorem 2. The dynamical systems formed by degree two polynomials are in one-to-one correspondence with the one-parameter family of polynomials $x^2 + c$.

2 Degree Three Polynomials

What happens if we consider degree three polynomials? To solve the Fundamental Problem, we may as well try the same approach that worked for degree two polynomials. Consider a generic degree three polynomial

$$F(x) = Ax^3 + Bx^2 + Cx + D \quad \text{with} \\ A, B, C, D \in \mathbb{Q} \text{ and } A \neq 0$$

and conjugate by an arbitrary $\alpha(x) = sx + t$. We get

$$F^\alpha = As^2x^3 + (3Ast + Bs)x^2 + (3At^2 + 2Bt + C)x \\ + \frac{At^3 + Bt^2 + Ct + D - t}{s}.$$

Again, we have two degrees of freedom (s and t) and can choose $s = \frac{1}{\sqrt{A}}$ and $t = \frac{-B}{3A}$ to get

$$F^\alpha = x^3 + C'x + D'$$

for some new constants C' and D' . However, what if A is not a perfect square? Then if we consider all pairs $C', D' \in \mathbb{Q}$, then we will not get all degree three polynomials defined over $\mathbb{Q}$. This is the first time that the “nice” requirement has really come to the forefront. Why would we care about staying rational (defined over $\mathbb{Q}$)? The simplest answer lies in the perspective of number theory versus geometry. The curve $x^3 + y^3 = z^3$ as a geometric object has lots of points defined over the complex numbers $\mathbb{C}$. However, Fermat’s Last Theorem says that the only rational points are the trivial solutions (at least one coordinate is 0). As you know from the mathematics surrounding Fermat’s Last Theorem, questions surrounding rationality can be very subtle and deep. If we want to examine these kinds of subtle rationality questions for dynamical systems, then we must be very careful about choosing a form to represent these dynamical systems that preserve rationality. Consequently, we need another way to represent degree three polynomials.

2.1 Fixed Points and Multipliers

To find another way to represent degree three polynomials, we will use the fixed points and their multipliers.

Definition We say that x is fixed by f if $f(x) = x$.

For a degree three polynomial, there are three fixed points: the solutions to $f(x) - x = 0$.

Example For $f(x) = x^3 - 6x^2 + 12x - 6$ we have

$$f(x) - x = (x - 1)(x - 2)(x - 3)$$

so the three fixed points are $x \in \{1, 2, 3\}$.

Definition For each fixed point x of f , we define the multiplier at x to be

$$\lambda_x = f'(x),$$

where f' is the derivative of f .

Example Continuing Example 2.1, we see

$$f'(x) = 3x^2 - 12x + 12$$

so that

$$\begin{aligned}\lambda_1 &= f'(1) = 3 - 12 + 12 = 3 \\ \lambda_2 &= f'(2) = 12 - 24 + 12 = 0 \\ \lambda_3 &= f'(3) = 27 - 36 + 12 = 3.\end{aligned}$$

We will need a fact about the set of multipliers that we will state without proof.

Theorem 3. *The set of fixed point multipliers is invariant under conjugation. In other words, the set of fixed point multipliers of f and f^α are the same for any linear fractional transformation α .*

Be careful, the multipliers are fixed as a set, not as individual values. It is possible that conjugation permutes the multipliers. Consequently, we wish to find values that are independent of the conjugation. Specifically, we define three values σ_1, σ_2 , and σ_3 that do not change when the multipliers are permuted. This type of value is called *symmetric* in $\{\lambda_1, \lambda_2, \lambda_3\}$.

Definition Let f be a degree three polynomial. Let λ_1, λ_2 , and λ_3 be the multipliers of the three fixed points of f . Define the following three values

$$\begin{aligned}\sigma_1 &= \lambda_1 + \lambda_2 + \lambda_3 \\ \sigma_2 &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \\ \sigma_3 &= \lambda_1\lambda_2\lambda_3.\end{aligned}$$

In fancy language, we would say that σ_1, σ_2 , and σ_3 are invariants of the moduli space of degree three polynomials.

The functions σ_1, σ_2 , and σ_3 are called the elementary symmetric polynomials and represent the connection between the roots of a polynomial and its coefficients.

Example Notice that for $f(x) = x^2 + c$, there are two fixed points

$$x = \frac{1 \pm \sqrt{1 - 4c}}{2}$$

with multipliers

$$\lambda_i = 1 \pm \sqrt{1 - 4c} \quad \text{for } i = 1 \text{ or } 2.$$

So the invariants in this case are

$$\begin{aligned}\sigma_1 &= \lambda_1 + \lambda_2 = 2 \\ \sigma_2 &= \lambda_1\lambda_2 = 4c.\end{aligned}$$

So we could re-write the set of all degree two polynomial dynamical systems as

$$x^2 + \frac{\sigma_2}{4}.$$

What remains to be done is to write the dynamical systems formed from degree three polynomials in terms of the invariants σ_1, σ_2 , and σ_3 . While possible, this is not easy.

Theorem 4. ([2]) Define the curve $\mathcal{C} : 4\sigma_1^3 - 36\sigma_1^2 + 81\sigma_1 + 27\sigma_3 - 54 = 0$. Then all dynamical systems from degree three polynomials are determined as follows.

$$\left\{ \begin{array}{ll} \frac{1}{(16\sigma_1^4 - 144\sigma_1^3 + 468\sigma_1^2 - 648\sigma_1 + 324) ((4\sigma_1^3 - 36\sigma_1^2 + 81\sigma_1 + 27\sigma_3 - 54) z^3 + (8\sigma_1^4 - 96\sigma_1^3 + 378\sigma_1^2 + 54\sigma_1\sigma_3 - 594\sigma_1 - 162\sigma_3 + 324) z^2 + (36\sigma_1^2\sigma_3 - 216\sigma_1\sigma_3 + 324\sigma_3) z + (8\sigma_1^3\sigma_3 - 72\sigma_1^2\sigma_3 + 216\sigma_1\sigma_3 - 216\sigma_3))} & (\sigma_1, \sigma_3) \notin \mathcal{C} \\ \frac{(-4\sigma_1\sigma_3 + 24\sigma_3)z^3 + (6\sigma_1^2 - 45\sigma_1 - 9\sigma_3 + 54)z^2 + (-2\sigma_1^2 + 33\sigma_1 - 45)z - 2\sigma_1 - 6}{9\sigma_1 - 27} & (\sigma_1, \sigma_3) \in \mathcal{C} \\ \begin{array}{l} z^3 \\ z^3 + z \\ z^3 + \frac{3}{2}z \end{array} & \begin{array}{l} (\sigma_1, \sigma_3) = (6, 0) \\ (\sigma_1, \sigma_3) = (3, 1) \\ (\sigma_1, \sigma_3) = (\frac{3}{2}, 0) \end{array} \end{array} \right.$$

Theorem 4 provides a solution to the Fundamental Problem for degree three polynomials. It gives a two-parameter family of polynomials representing every distinct degree three polynomial dynamical system exactly once. The form may look complicated, but this form preserves rationality, so it is “nice” for rationality questions.

There are alternative ways to solve the Fundamental Problem for degree three polynomials. One alternate solution appears in [1], where they use that every degree three polynomial is conjugate to one of two forms

$$ax^2 + bx + 1 \text{ or } ax^2 + bx.$$

However, these do not uniquely determine dynamical systems of degree three polynomials since there can be distinct representatives that are conjugate.

Example The two maps

$$\begin{aligned} f(x) &= x^3 + x \\ g(x) &= 4x^3 + x \end{aligned}$$

are conjugate via $\alpha(x) = 2x$.

These two forms are less complicated than the forms in Theorem 4, but they only partially solve the Fundamental Problem, whereas Theorem 4 provides a full solution.

Open Problem 1: Can you resolve the Fundamental Problem for polynomials of degrees larger than three?

Open Problem 2: Can you resolve the Fundamental Problem for other classes of dynamical systems? (Degree two rational functions are solved in [5].)

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What is happening in Ritter?

Third meeting of the Saint Louis Academy of Mathematical Sciences

By Hugo Panzo, Assistant Professor in the Department of Mathematics and Statistics at Saint Louis University

The Saint Louis Academy of Mathematical Sciences had its 3rd annual meeting on Friday, November 18th in the Pere Marquette Gallery on the campus of Saint Louis University. The keynote speaker for the evening was Stefan Steinerberger of the University of Washington in Seattle. This year's meeting was well-attended by local mathematicians, statisticians, and data scientists and was a welcome return to normality after the pandemic-forced postponement of the 2021 edition. Guests hailed from across the Greater St. Louis metropolitan area and represented SLU, Washington University in St. Louis, University of Missouri–St. Louis, Southern Illinois University Carbondale, Harris–Stowe State University, and Missouri University of Science and Technology.



The inaugural meeting of the Saint Louis Academy of Mathematical Sciences, or SLAMS for short, took place in November of 2018. The brainchild of WashU and SLU math professors John McCarthy and Bryan Clair, SLAMS traces its origins back to a lunch meeting between the two department chairs back in the summer of 2018. In the words of McCarthy, SLAMS was conceived as a way to “foster contacts between the departments at WashU, SLU, UMSL, and beyond,

and also to bring in people who are not actually in a mathematics department but still are interested in mathematics in its broad sense.” Previous keynote speakers were Fields Medal winner Efim Zelmanov of UC San Diego and Michael Christ of UC Berkeley.



The evening started with Stefan Steinerberger's keynote address, which was titled *Revisiting the Idea of Boundary*. Steinerberger, an up-and-coming mathematician known for both his incredible research productivity and his entertaining math talks, began by recounting the legend of Queen Dido and the founding of Carthage. At the heart of this story is the isoperimetric problem, a famous mathematical problem whose intuitively obvious solution required nearly 3000 years and surprisingly sophisticated methods to prove. Indeed, as Steinerberger's talk would go on to demonstrate, the isoperimetric problem continues to inspire research in the mathematical sciences to this day.

Once Steinerberger had concluded his talk and fielded several questions from the audience, guests took advantage of a break in the schedule to mingle by the bar and at their tables while waitstaff began the evening's dinner service. The conversation ranged from mundane introductions to animated discussions of the talk. Colleagues who hadn't seen each other in years quickly reacquainted themselves and began discussing their teaching schedules and research programs. Some guests made plans for future collaboration, while others were invited to give colloquium talks at neighboring institutions. As the event came to a close, it was clear

to all in attendance that this year's SLAMS meeting had been a resounding success. It is safe to say that next year's meeting and its yet-to-be-determined location and speaker will be eagerly anticipated by the St. Louis mathematical community.



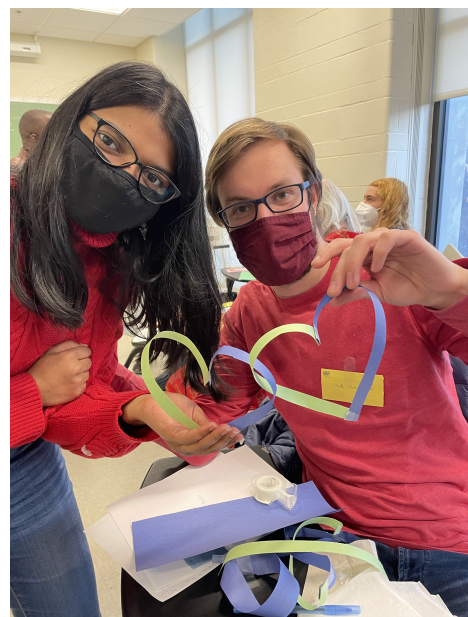
Math + CS Club

By Kieran Favazza, Undergraduate Student at Saint Louis University

Reflecting the spirit of this publication and of the Department of Mathematics and Statistics, the Saint Louis University Math and Computer Science Club is a welcoming and inclusive environment for all lovers of math and learning. The club meets on Wednesdays from 3:30 to 5:00 PM in the lobby of Ritter Hall, and each meeting starts with snacks, drinks, pizza, and socializing. Faculty and students of all majors relax, share their stories, and then engage in an entertaining math activity. This year, the club has had such notable activities as taco parties, building complex origami shapes, Math Jeopardy!, Math Taboo, board games, and presentations from the math faculty. Each one of these has been enormously fun, almost always featuring a group project that enables the members to take something interesting and mathematical home with them.



When the club members aren't making dodecahedra out of a few sheets of paper, fractals out of ordinary objects, or representations of strange math concepts with just colors and paper, they engage their competitive and inquisitive spirits with games and competitions, uniting everyone in that burning excitement that learning can bring. Each day, students and faculty learn more about themselves and each other, reminding us that math is a pursuit meant to be undertaken alongside our friends. The Math and Computer Science Club at SLU reminds us of how fun and silly math can be - not always something we must pull our hair out over - so please join! Please come enjoy the welcoming company of curious learners, the pizza, and the math the next time you can spare a Wednesday afternoon!



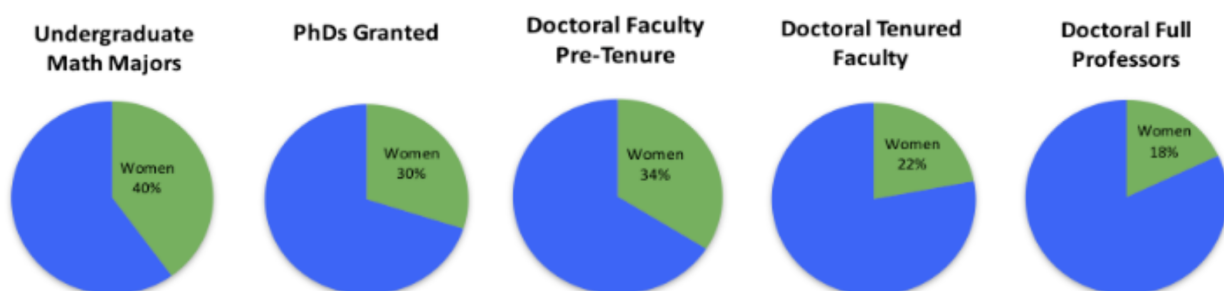
SLU AWM Student Chapter

By Mary Silvergate, Masters Student at Saint Louis University

The Saint Louis University (SLU) chapter of the Association for Women in Mathematics has grown exponentially in just a year and a half. We have many women or gender-nonconforming mathematics students but did not have an active organization promoting diversity in mathematics until the fall semester of 2021. Since founding the chapter, we have seen students from across multiple disciplines and multiple backgrounds join our weekly meetings, including mathematicians, geologists, engineers, and chemists from all genders. Our bi-weekly meetings are dedicated to the advancement of women and other gender minorities in mathematics, and we regularly see members of all genders involved in our organization. Beyond recruiting, we have held community-building events, talks led by women in varying fields of mathematics, and we have created mentorship opportunities for women and other gender minorities within multiple academic departments.



As a new organization at SLU, one of our first goals was to establish a supportive community within the mathematics department, particularly for women and other gender minorities. To form this community, we have specifically focused on facilitating cross-generational mentorship. Our meetings regularly include undergraduate, graduate, and faculty members interacting and giving each other advice. From the beginning, we designed our board intentionally to include both a Graduate President and an Undergraduate President. By doing this, we have encouraged relationship-building across the graduate and undergraduate schools and ensured that the chapter will continue beyond the involvement of its founding members. We wish to reverse the trend seen in the attached infographic from the National Association for Women in Mathematics. Approximately 40% of undergraduate math majors are women, but only 18% of doctoral full professors are women. We hope that by providing community and mentorship opportunities, those missing 22% feel more comfortable choosing a career in mathematics.

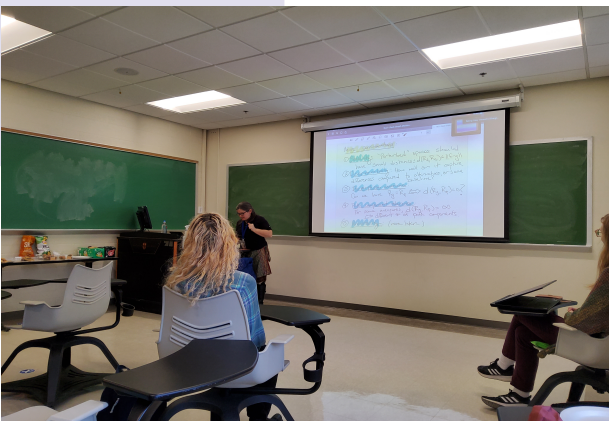


Data from the 2015-2016 Annual Survey by the American Mathematical Society

In addition to facilitating networking, we have hosted mathematically themed de-stressing events, such as a screening of the motion picture *Hidden Figures*, as well as pre-finals mathematical coloring and crafts. Many majors and genders have shown up to these events with fidelity, not only mathematics majors or women. Several of our events, such as our speaker series and de-stress events, are repeated events and have thus encouraged a communal environment that includes time for socializing before and after meetings with old friends and new members.



Our most successful recurring event has been our ongoing speaker series by women in the department regarding their mathematical research. In 2021-2022 alone, we hosted four speakers, with each talk having over 30 attendees from various departments. Prior to starting this speaker series, there were limited opportunities for undergraduates to learn about modern mathematical research, let alone by women in the department. We have showcased a diversity of work happening in our department, from mathematical physics to combinatorics to frame theory. Additionally, we have hosted women from other departments such as Dr. Erin Chambers, a professor from the Department of Computer Science who conducts research in computational geometry and topology in her field, and Dr. Shaneeka Favors-Welch, a postdoctoral scholar in the Department of African American Studies and statistical educational researcher.



We plan to build upon the strong foundation we have laid as a club over the past year and a half to build an even stronger community among women in mathematics and mathematics-adjacent fields in the future. We have grown the SLU Association for Women in Mathematics club from zero members to about twenty regular members in a short period of time, and we are excited to discover how much more growth we will have in the future. Hopefully, we will see you at one of our future meetings!



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or Thursdays and Sunday evenings
from 6-9pm via Zoom

$$\frac{t_{9x-2} Q''}{\sqrt{11x3}}$$

$$e = cc$$

$$y = 2x^3$$

$$y = \frac{\Delta x}{\Delta^2}$$

$$\int (x \pm a^4)$$

$$e = 2.7g$$

$$S = \int_{t=2}^{10} \dots$$

$$+a^2$$

$$\frac{b \pm \dots}{\sqrt{2}}$$

$$\sin a = \dots$$

$$S_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

